



AMALDI
RESEARCH CENTER

The New Era of Gravitational Waves



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Schedule for the week

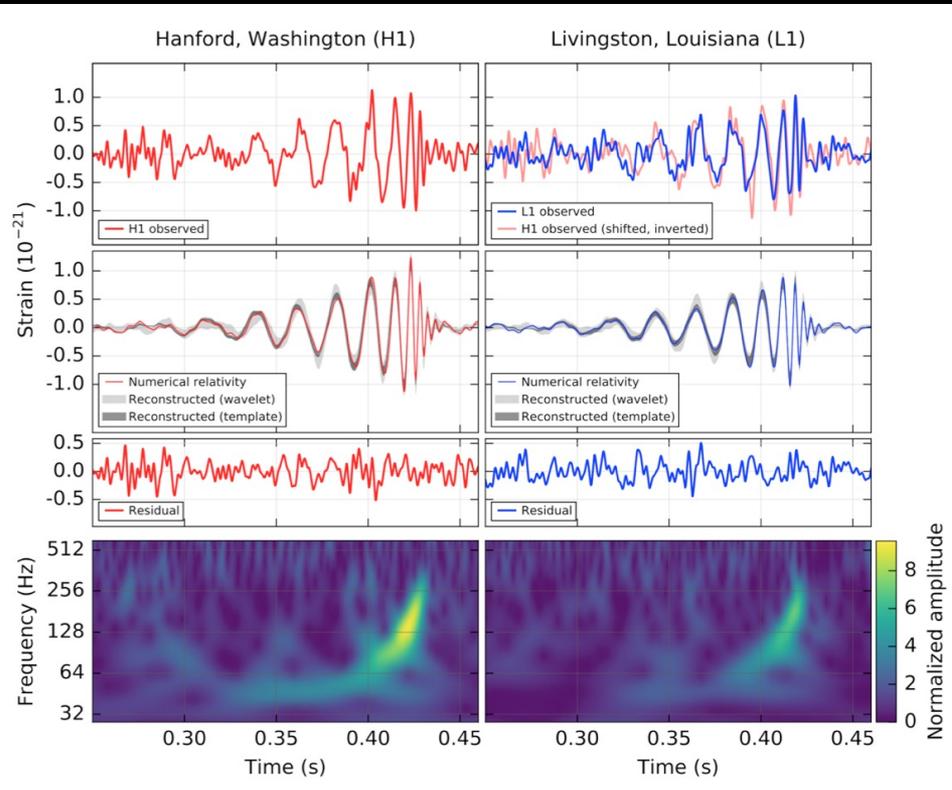
- ***Monday:*** Gravitational Waves: basic and data analysis
- ***Tuesday:*** Interferometric detectors of Gravitational Waves
- ***Wednesday:*** 90 Gravitational Wave detections: what did we learn?
- ***Thursday:*** Multimessenger probes
- ***Hands-on session:*** Gravitational Wave Open Science Center



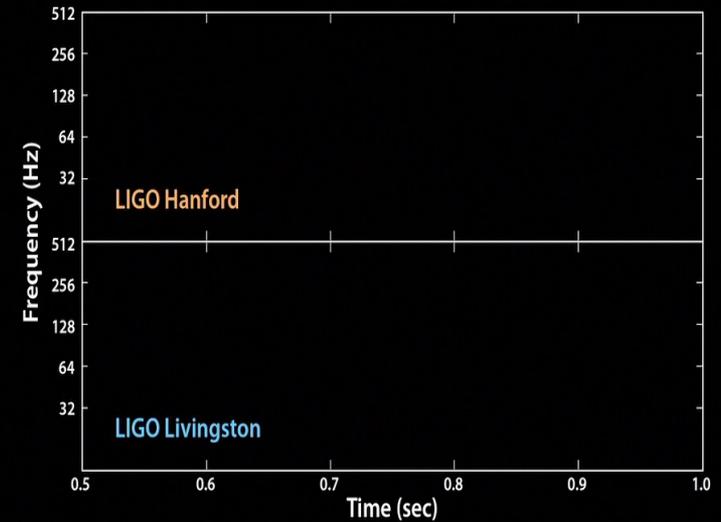
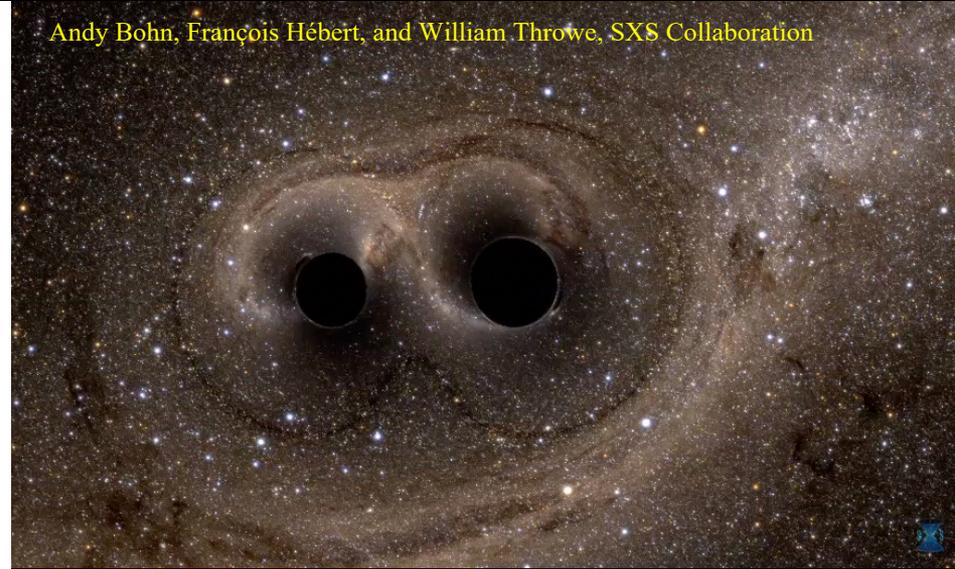
Outline

- Einstein and the General Relativity
- Gravitational Waves as solutions of Einstein field equations
- Experimental tests for the theory of Gravitation
- Hulse-Taylor binary pulsar
- Gravitational Wave sources
- Matched Filter
- Glitches
- Un-modeled search

GW150914: The First Binary Black Hole Merger



Andy Bohn, François Hébert, and William Throwe, SXS Collaboration



Abbott, et al. ,LIGO Scientific Collaboration and Virgo Collaboration, "Observation of Gravitational Waves from a Binary Black Hole Merger" [Phys. Rev. Lett. 116, 061102 \(2016\)](#)



Gravitational wave science

The **Study of gravitational waves** is at the *frontiers of science* in at least four different fields:

- **General Relativity (GR) – physics at the extremes: strong (non-linear) gravity, relativistic velocities**
- **Astrophysics of compact sources – neutron stars, black holes, the big bang – the most energetic processes in the universe**
- **Interferometric gravitational wave detectors – the most precise measuring devices ever built**
- **GW data analysis – the *optimal* extraction of the weakest signals possible out of noisy data.**



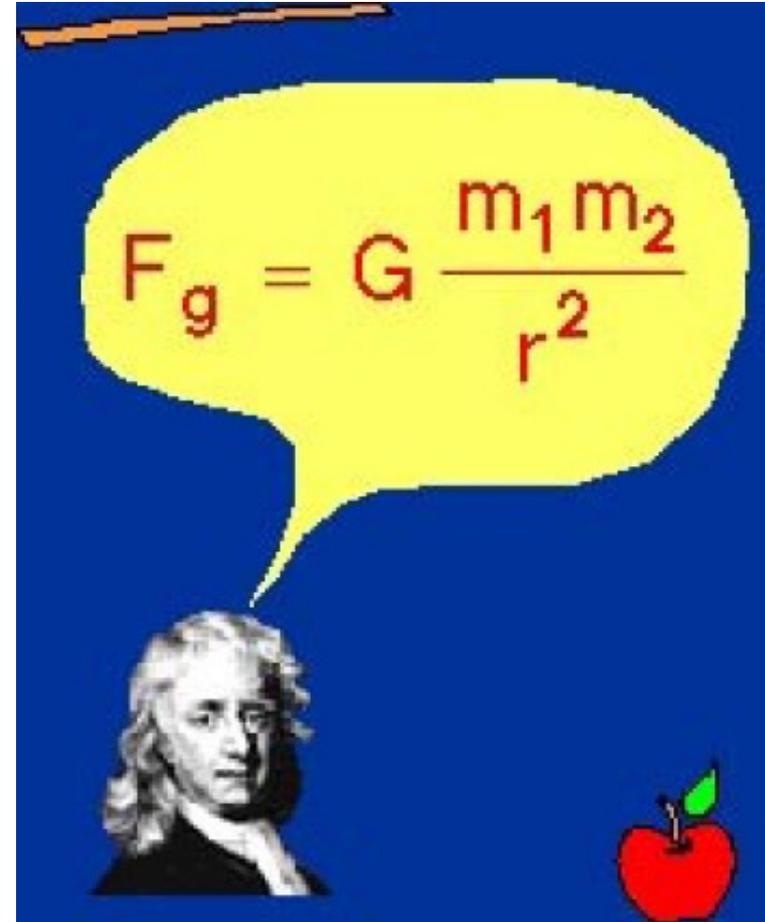
Einstein's view of gravity: The General Theory of Relativity

- Starting in 1915, Albert Einstein began the development of a new theory of gravity.
- The basic idea is that gravity is not a force, but rather a manifestation of the curvature of space-time.
- Space and time aren't just a simple backdrop to the world, but have properties of their own. In particular, they can be “curved”, which means that matter can be prevented by the properties of space-time from moving uniformly in a straight line.
- Space-time curvature is caused by mass.

Thus, **General Relativity** embodies the idea of gravity, and even “explains” it.

Newtonian Gravity

- Three laws of motion ($F=ma$) and law of gravitation (centripetal force) disparate phenomena
 - » Eccentric orbits of comets
 - » Cause of tides and their variations
 - » The precession of the earth's axis
 - » The perturbation of the motion of the moon by gravity of the sun
- Solved most known problems of astronomy and terrestrial physics
 - » Work of Galileo, Copernicus and Kepler unified.
- Gravitational fields are static (or slowly changing) the force acts over large distances, “instantaneously”



Einstein and relativity

It all starts with Einstein!

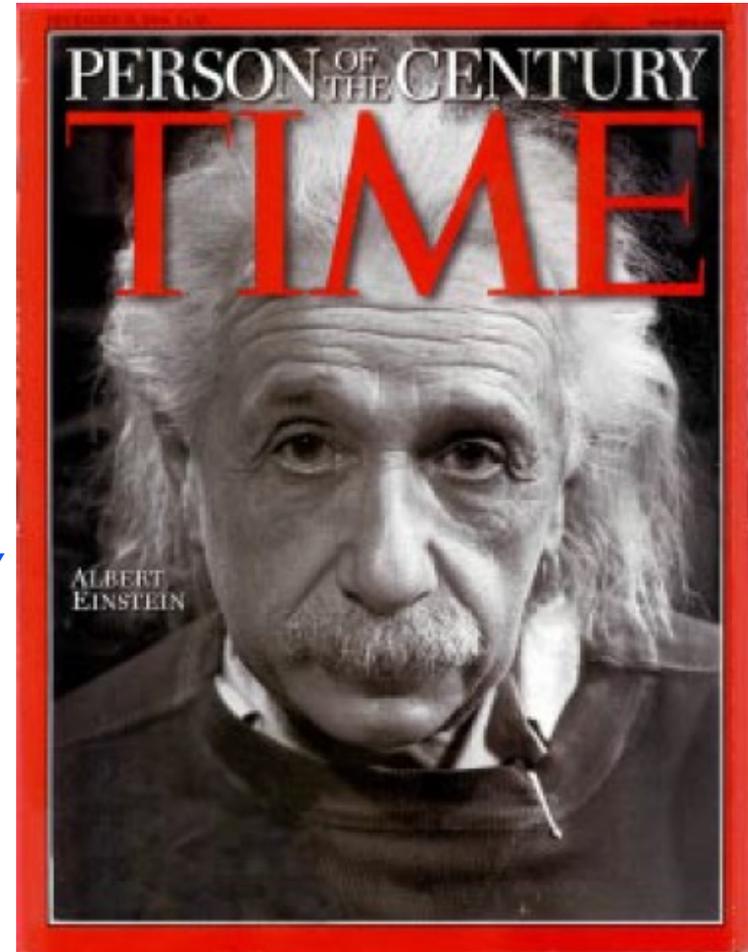
- Special relativity (1906)

- » Distances in space and time change between observers moving relative to one another, but the space-time interval remains invariant:

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$$

- » space + time \rightarrow 4D space-time *geometry*
- » Energy and momentum form a 4D vector with invariant (rest) mass:

$$(m_0c^2)^2 = E^2 - (pc)^2 \quad (\text{or } E = mc^2)$$



Space-time geometry

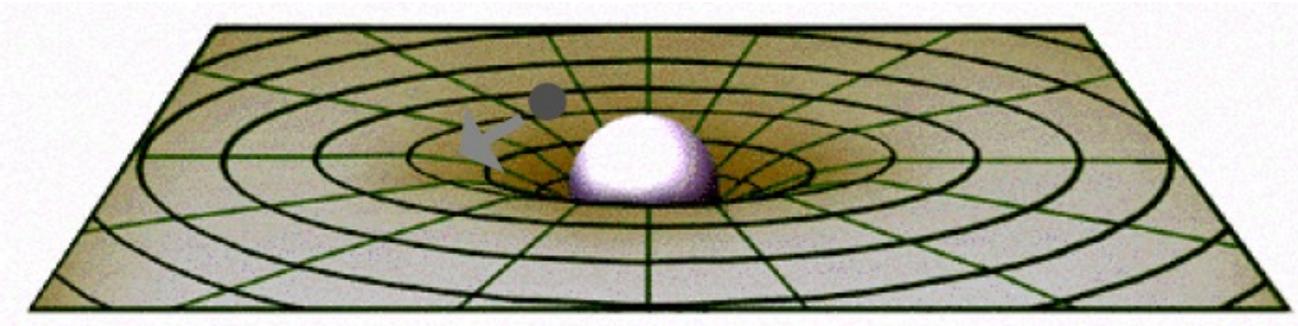
Relativity and space-time geometry:

- Discards concept of absolute motion; instead treats only relative motion between systems
- Space and time no longer viewed as separate; rather as four dimensional space-time
- Gravity described as a warpage (curving) of space-time, not a force acting at a distance



Warped space-time: Einstein's General Relativity (1916)

- A geometric theory of gravity
 - » Gravitational acceleration depends only on the geometry of the space that the “test mass” occupies, not any properties of the test mass itself
 - » For gravity (as opposed to all other forces), motion (acceleration) depends only on location, not mass
- Image space as a stretched rubber sheet.
- A mass on the surface will cause a deformation
- Another mass dropped onto the sheet will roll toward that mass
- Einstein theorized that smaller masses travel toward larger masses, not because they are “attracted” by a mysterious force, but because the smaller objects travel through space that is warped by the larger object.





Strength of gravitational force

Interaction	Strength	Acts on	Charge	Carried by	theory
Strong nuclear	10	Quarks	Color	Gluons (g) (massless)	QCD
Electromagnetic	10^{-2}	Charged particles	Electric charge	Photon (γ) (massless)	QED
Weak nuclear	10^{-13}	Quarks, leptons	“flavor” charge	W^+ , W^- , Z^0 (massive)	QFD
Gravitational	10^{-40}	All particles	Mass	Graviton(G) (massless)	GR...?

Gravitational force is very weak!

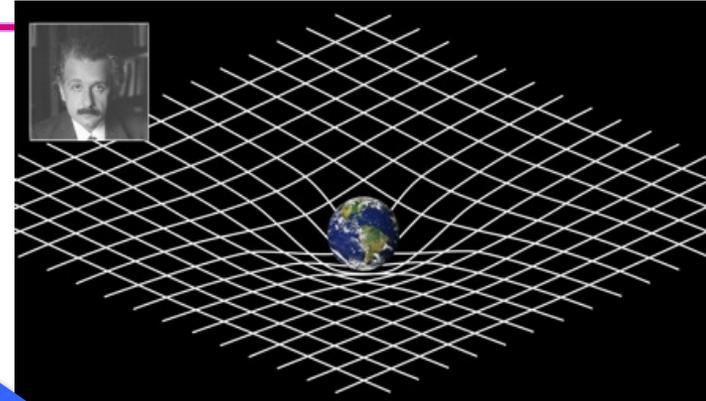
But at large scales (planets, stars, galaxies, universe) it dominates.

General Relativity and Gravitational Waves

Einstein field tensor

Stress-energy-momentum tensor

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



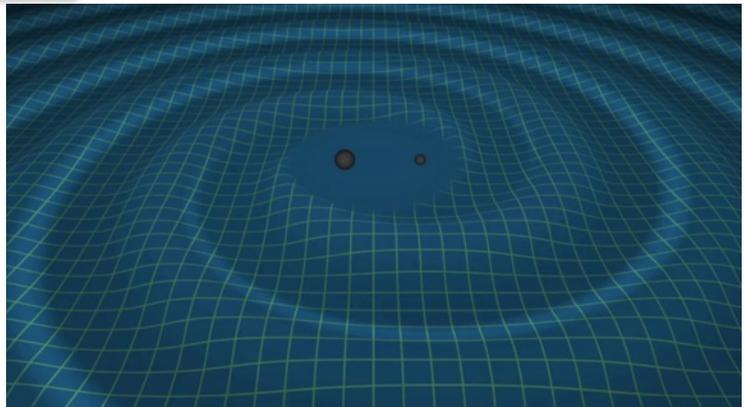
General Relativity: Einstein Field Equations

Weak field approximation - space-time is slightly perturbed from flat space-time:

$$g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$$

Free Space: $T_{\mu\nu} = 0$

$\sim 10^{-43}$



Wave equation for $h_{\mu\nu}$!

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) h_{\mu\nu} = 0$$

A. Einstein, *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften* (Berlin, 1916), 688696; *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften* (Berlin, 1918), 154167.

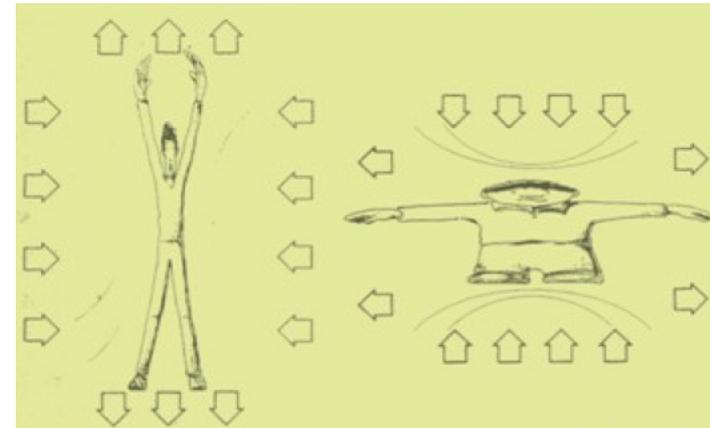
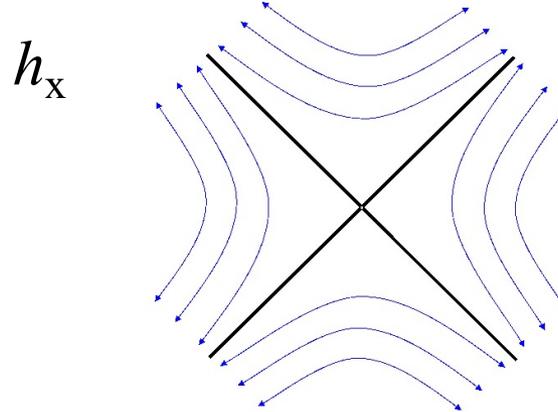
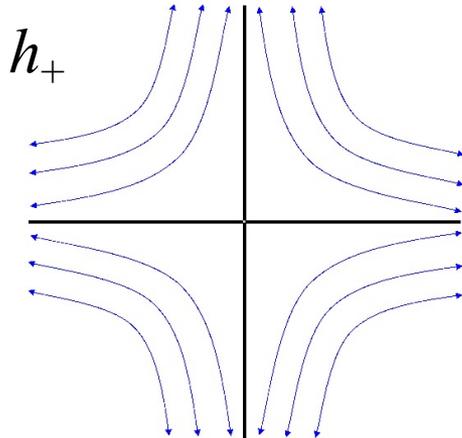
Gravitational Waves

Solution for an outward propagating wave in z-direction:

$$h(t, z) = h_{\mu\nu} e^{i(\omega t - kz)} = h_+(t - z/c) + h_x(t - z/c)$$

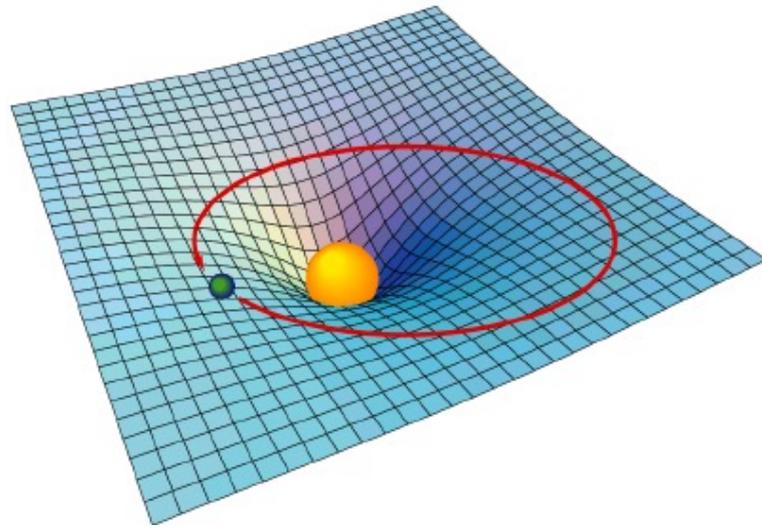
$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_x & 0 \\ 0 & h_x & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Physically, h is a strain: $\Delta L/L$



General relativity (1916)

- Space-time warps in response to the presence of matter, energy, motion.
- Motion of matter is determined by space-time curvature.
- For gravity (as opposed to all other forces), motion (acceleration) depends only on location, not mass.
- 16 coupled non-linear differential equations; analytical solutions in only the simplest of cases (spherical symmetry, static, etc).





Space-time geometry metric

- Einstein field tensor G is a function only of the space-time metric g which describes local geometry.
- Space-time interval ds (generalization of Pythagorean theorem to space-time):

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- g is space-time metric,
- η is flat space Minkowski metric,
- h is metric perturbation, for weak gravitational fields, components of $h \ll 1$.



Metric perturbation h

- In the weak-field limit ($h \ll 1$), Einstein's field equations can be linearized.
- In the “transverse traceless” (TT) gauge, they become a wave eqn for h (no matter sources):

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) h_{\mu\nu} = 0$$

- The metric perturbation is interpreted as a gravitational wave amplitude, travelling at the speed of light.
- Gravitational wave metric perturbations stretch and squeeze the space they pass through (strain amplitude).



Gravitational waves

- General relativity says almost the same thing, except the metric can be different.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

- The trick is to find a metric $g_{\mu\nu}$ that describes a particular physical situation.
- The metric carries the information on the space-time curvature that, in GR, embodies gravitational effects.



Gravitational waves

- Gravitational waves propagating through flat space are described by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- A wave propagating in the z -direction is described by

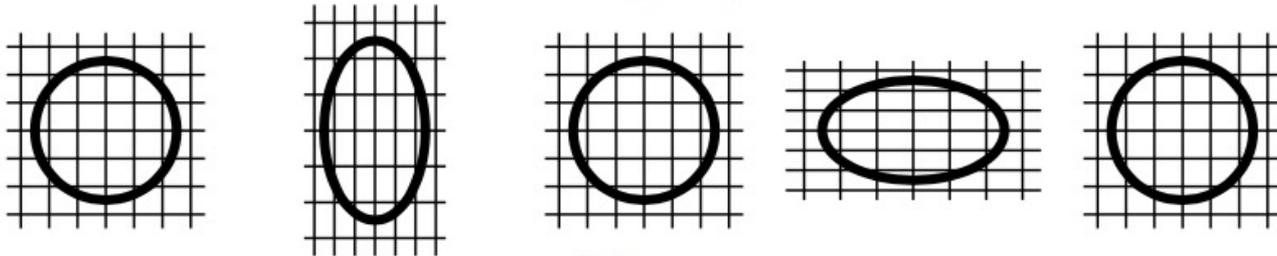
$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & b & -a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Two free parameters implies two polarizations

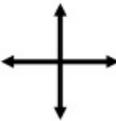
Gravitational waves

Gravitational waves are **deformations of space** itself, stretching it first in one direction, then in the perpendicular direction.

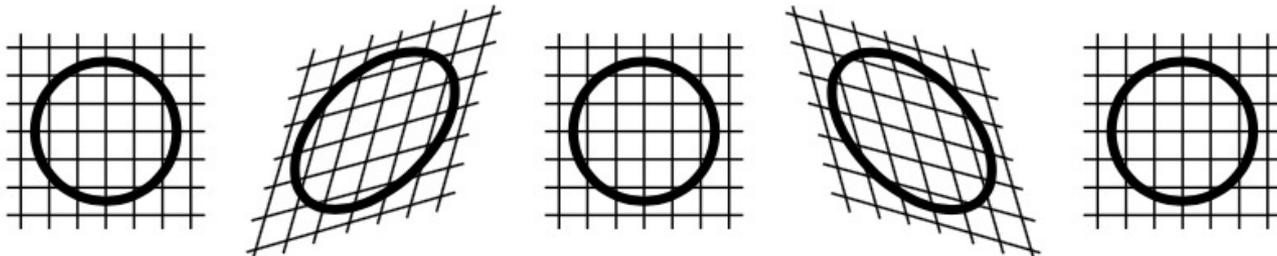
“Plus”
polarization



Time

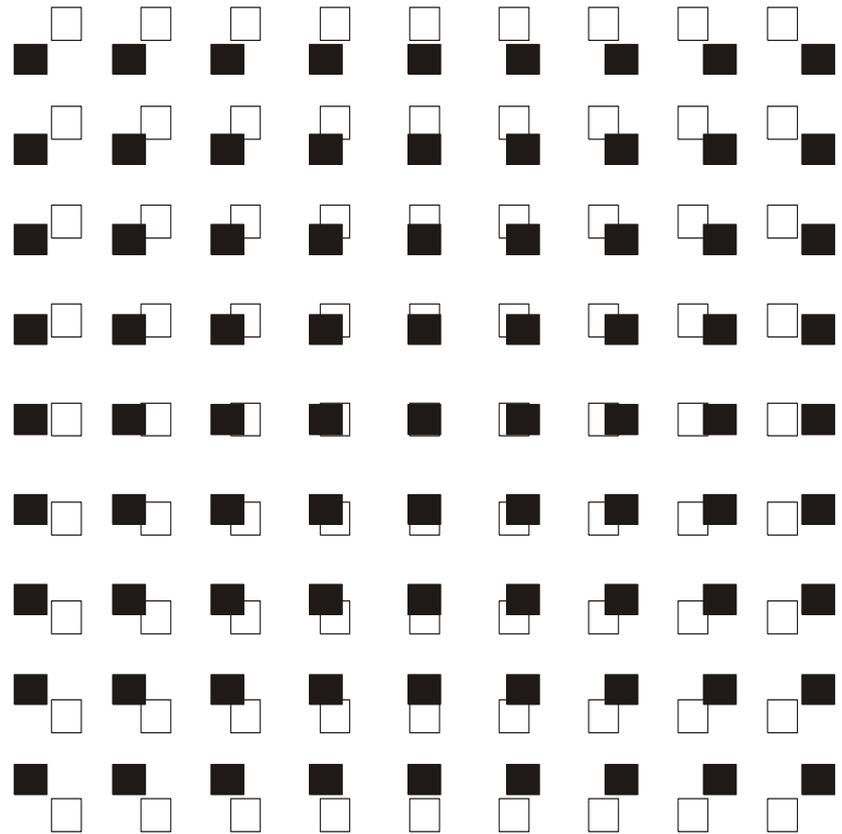


“Cross”
polarization



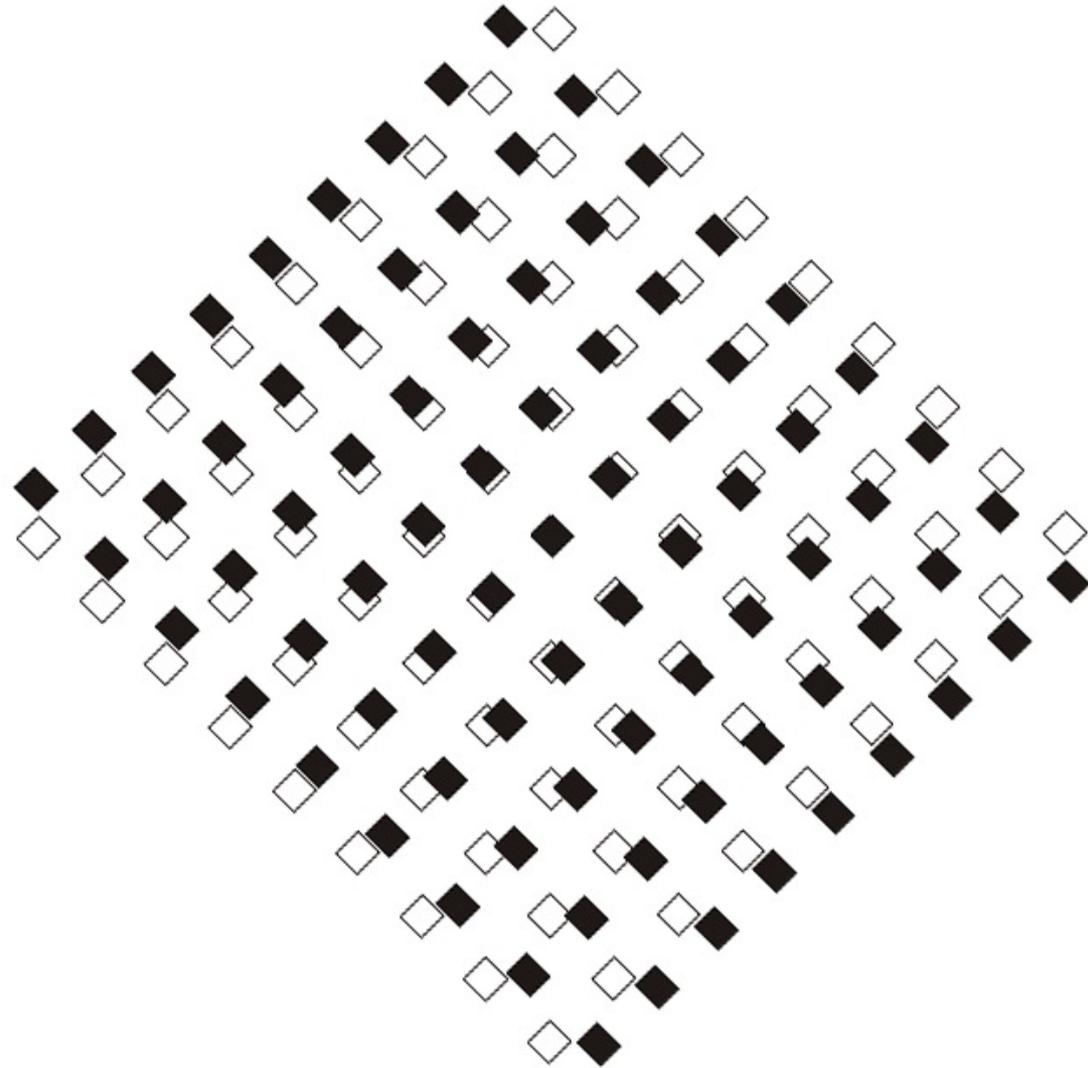
Plus polarization

$$\hat{h}_+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



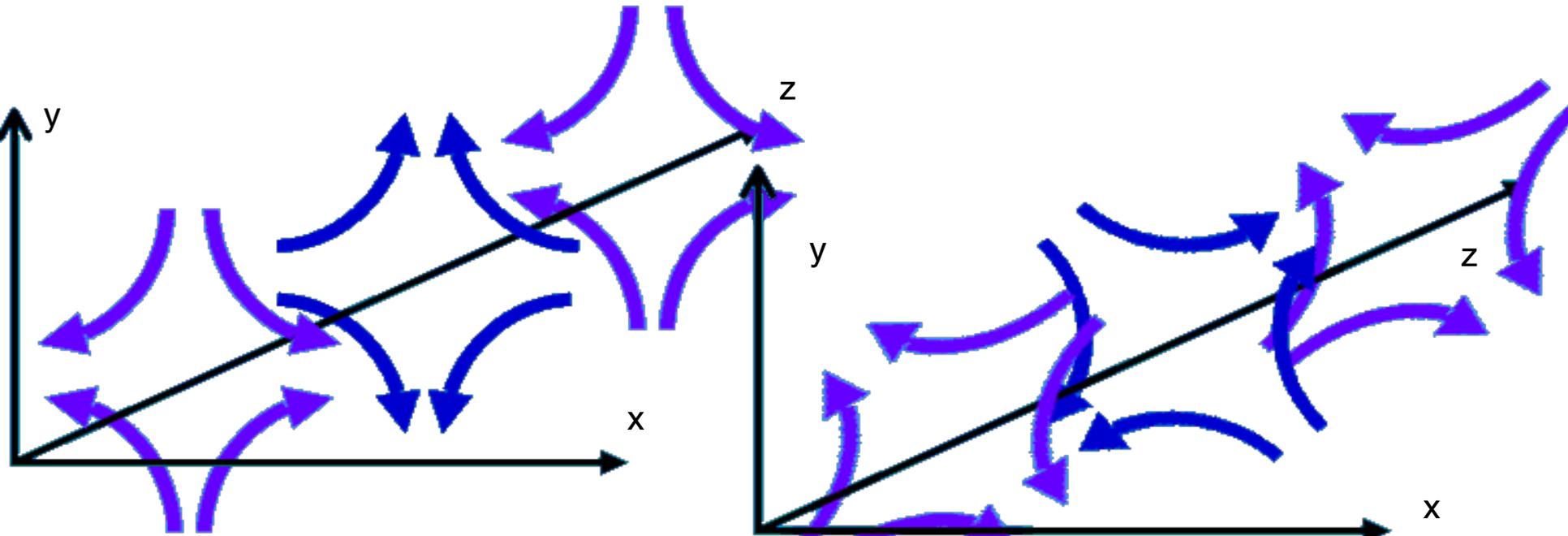
Cross polarization

$$\hat{h}_x = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



Gravitational wave basics

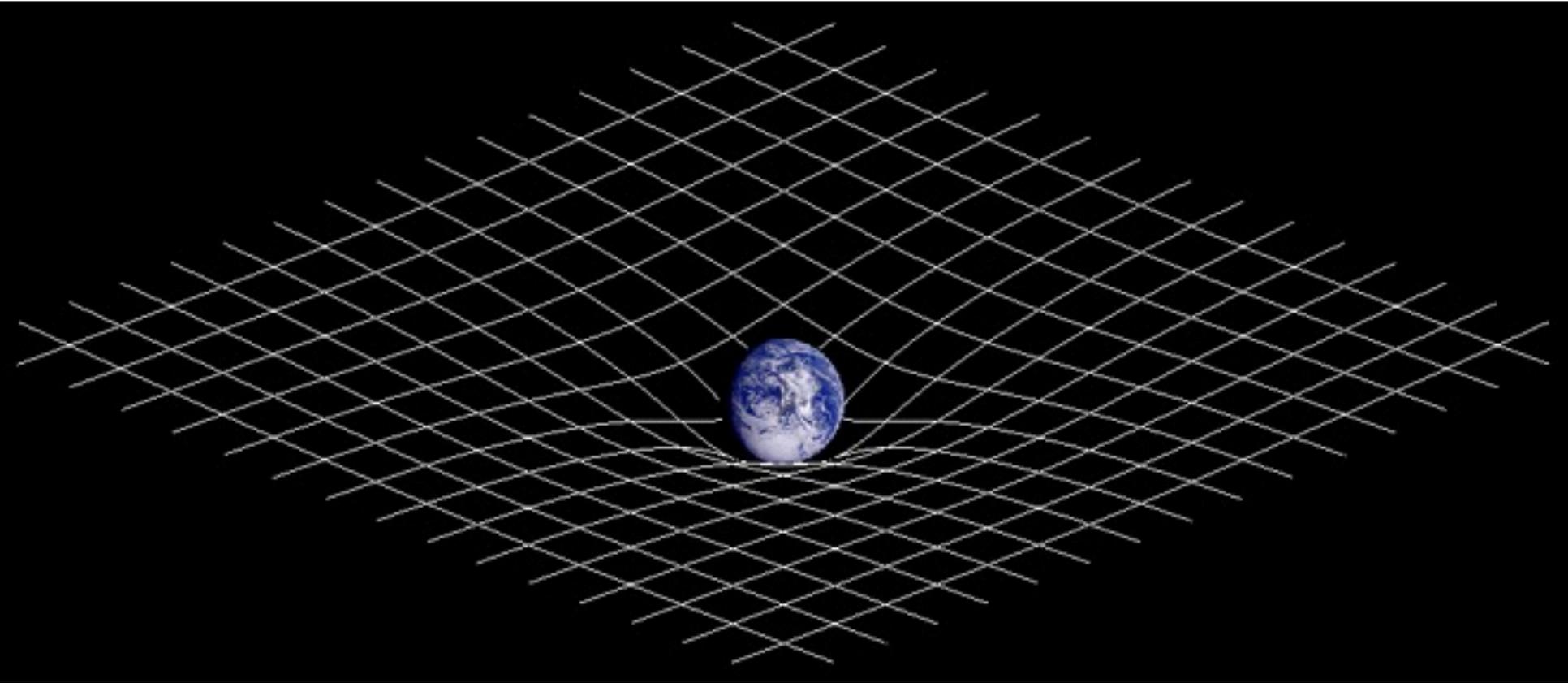
- The matrix shows that the arm length shifts in the x and y directions are of opposite signs (when one is compressed, the other is stretched), so the phase difference in the two arms thus produced would add coherently.
 - Linear algebra further tells us that the directions that receive the largest amount of stretching and compressing (x and y in our example above) are the eigen-directions of the matrix
- $$\begin{pmatrix} h_+ & 0 \\ 0 & -h_+ \end{pmatrix}$$
- or in other words the spatial transverse part of the matrix.





*“Matter tells space-time how to curve.
Space-time tells matter how to move.”*

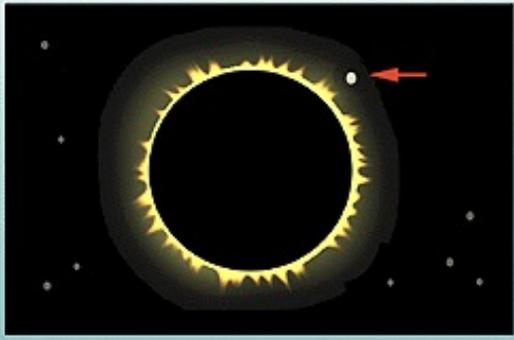
John Archibald Wheeler



Einstein's Theory of Gravitation

experimental tests

BENDING LIGHT

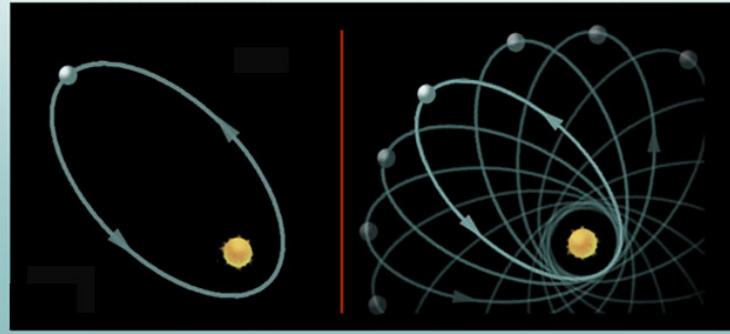


bending of light

As it passes in the vicinity of massive objects

First observed during the solar eclipse of 1919 by Sir Arthur Eddington, when the Sun was silhouetted against the Hyades star cluster

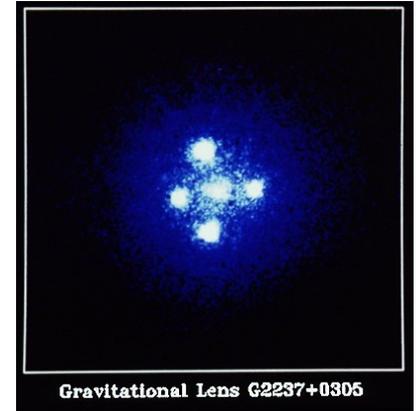
MERCURY'S ORBIT



Mercury's orbit

perihelion shifts forward twice Post-Newton theory

Mercury's elliptical path around the Sun shifts slightly with each orbit such that its closest point to the Sun (or "perihelion") shifts forward with each pass.



"Einstein Cross"

The bending of light rays gravitational lensing

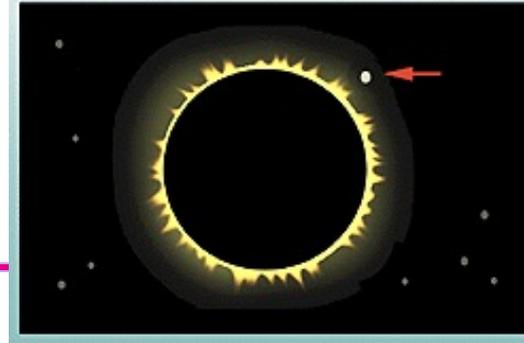
Quasar image appears around the central glow formed by nearby galaxy. Such gravitational lensing images are used to detect a 'dark matter' body as the central object



Einstein's Theory of Gravitation

experimental tests

BENDING LIGHT



- Predict the bending of light passing in the vicinity of the massive objects.
- First observed during the solar eclipse of 1919 by Sir Arthur Eddington, when the Sun was silhouetted against the Hyades star cluster.
- The measurements showed that the light from these stars was bent as it grazed the Sun, by the exact amount of Einstein's predictions.
- The light never changes course, but merely follows the curvature of space. Astronomers now refer to this displacement of light as gravitational lensing.

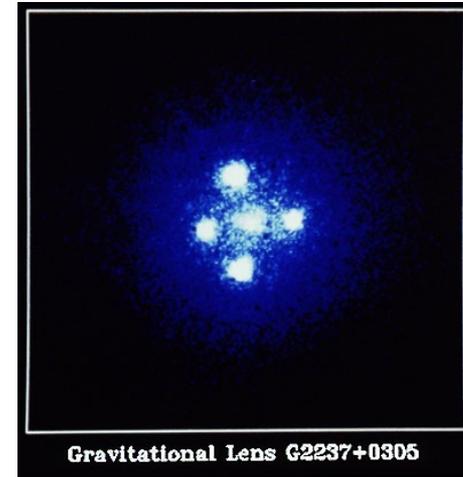
Einstein's Theory of Gravitation

experimental tests

“Einstein Cross”

The bending of light rays

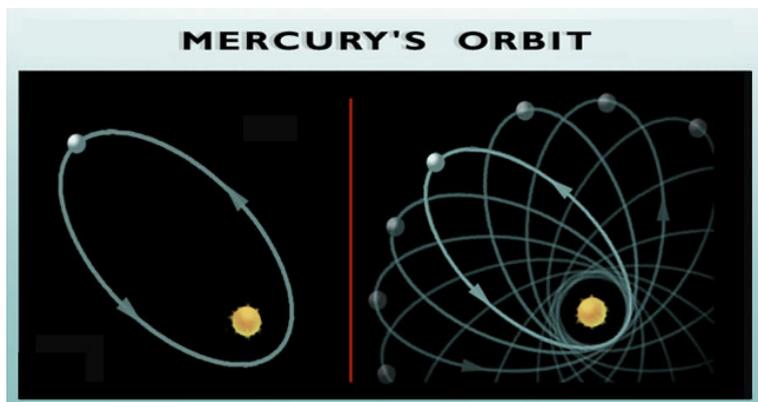
gravitational lensing



- Quasar (Q2237+0305) image appears around the central glow formed by nearby galaxy (ZW 2237+030). The Einstein Cross (Pegasus constellation) is only visible in southern hemisphere.
- In modern astronomy, such gravitational lensing images are used to detect a “dark matter” body as the central object.

Einstein's Theory of Gravitation

experimental tests



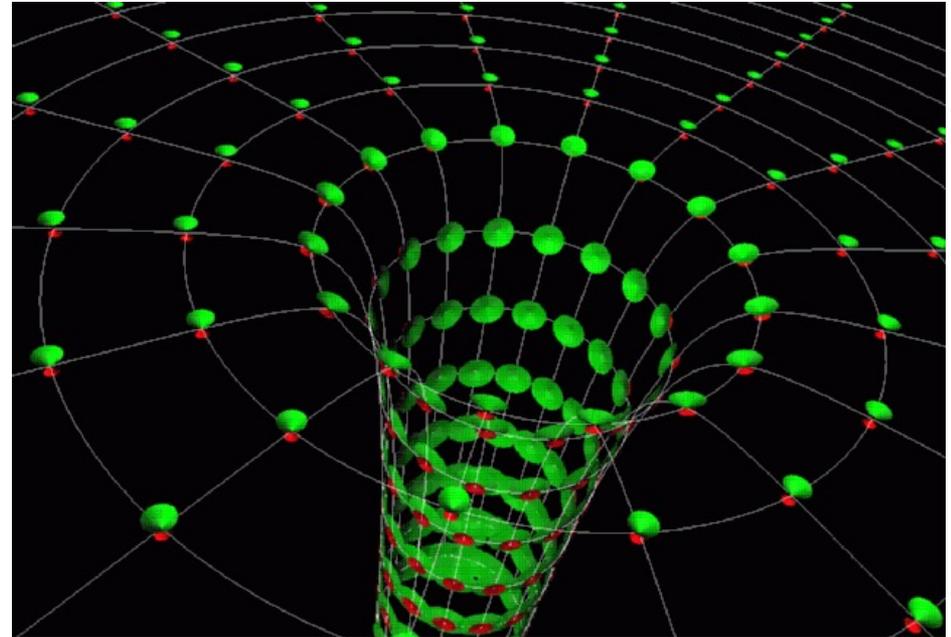
Mercury's orbit
perihelion shifts forward
twice Post-Newton theory

- Mercury's elliptical path around the Sun shifts slightly with each orbit such that its closest point to the Sun (or "perihelion") shifts forward with each pass.
- Astronomers had been aware for two centuries of a small flaw in the orbit, as predicted by Newton's law.
- Einstein predictions exactly matched the observation.

Strong-field

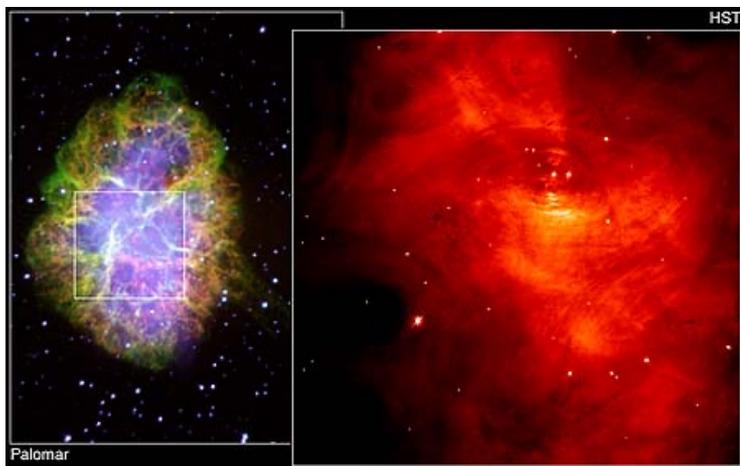


- Most tests of GR focus on small deviations from Newtonian dynamics (post-Newtonian weak-field approximation)
- Space-time curvature is a *tiny* effect everywhere except:
 - The universe in the early moments of the big bang
 - Near/in the horizon of black holes
- This is where GR gets *non-linear* and interesting!
- We aren't very close to any black holes (fortunately!), and can't see them with light or other EM radiation...



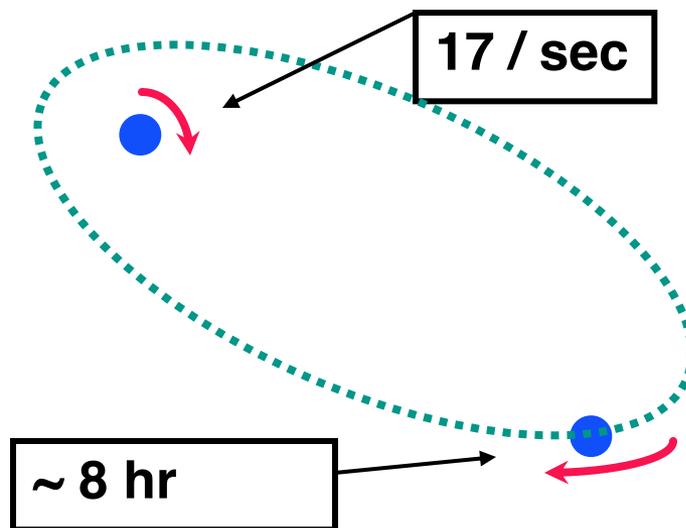
But we can search for (*weak-field*) gravitational waves as a signal of their presence and dynamics

Hulse-Taylor binary pulsar



Neutron Binary System

PSR 1913 + 16 -- Timing of pulsars



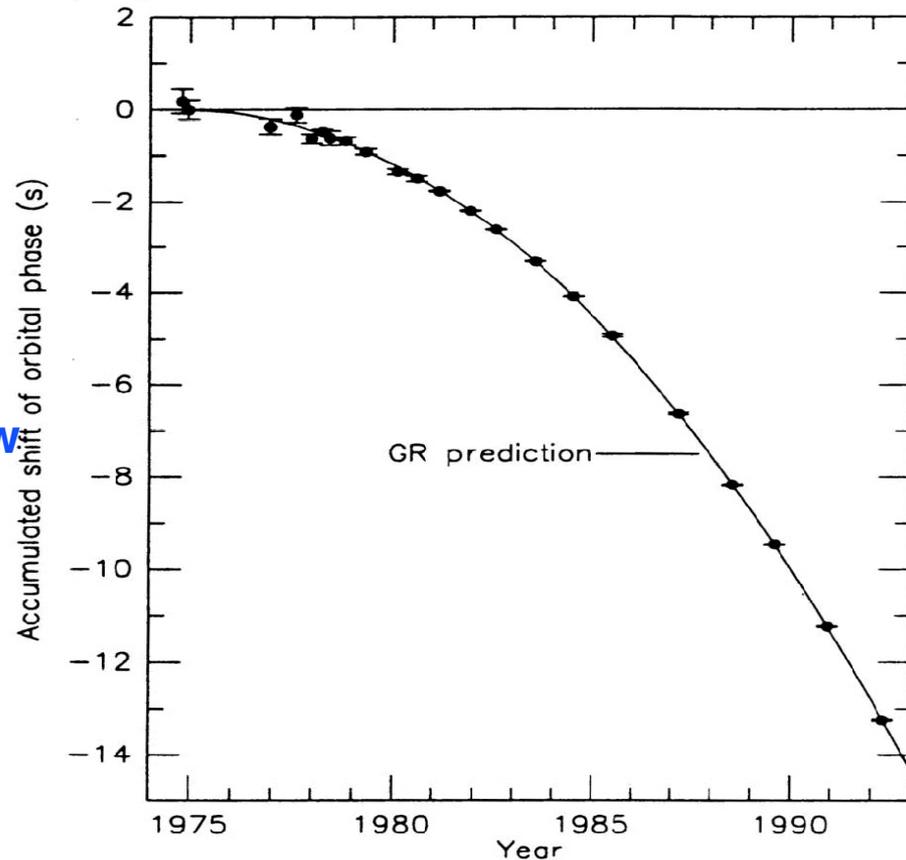
- A rapidly spinning pulsar (neutron star beaming EM radiation at us 17 x / sec)
- Orbiting around an ordinary star with 8 hour period
- Only 7 kpc away
- Discovered in 1975, orbital parameters measured continuously over 25 years!

GWs from Hulse-Taylor binary



emission of gravitational waves by compact binary system

- Period speeds up 14 sec from 1975-94
- Measured to ~50 msec accuracy
- Deviation grows quadratically with time
- Merger in about 300M years
(\ll age of universe!)
- shortening of period \Leftarrow orbital energy loss
- Compact system:
negligible loss from friction, material flow
- Beautiful agreement with GR prediction
- GW emission will be strongest near the end:
 - Coalescence of neutron stars!
- Nobel Prize, 1993



Basic Analysis Concepts

The output of a gravitational wave detector is a time series $s(t)$ that includes instrument noise $n(t)$ and the response to the gravitational wave signal $h(t)$:

$$s(t) = F^+(t)h_+(t) + F^\times(t)h_\times(t) + n(t).$$

The instrument response is a convolution of the antenna patterns F^+ , F^\times with the two gravitational wave polarizations h_+ , h_\times .

The information contained in the time series is usually represented in the Fourier domain as a strain amplitude spectral density, $h(f)$. This quantity is defined in terms of the power spectral density $S_s(f) = \tilde{s}^*(f)\tilde{s}(f)$ of the Fourier transform of the time series

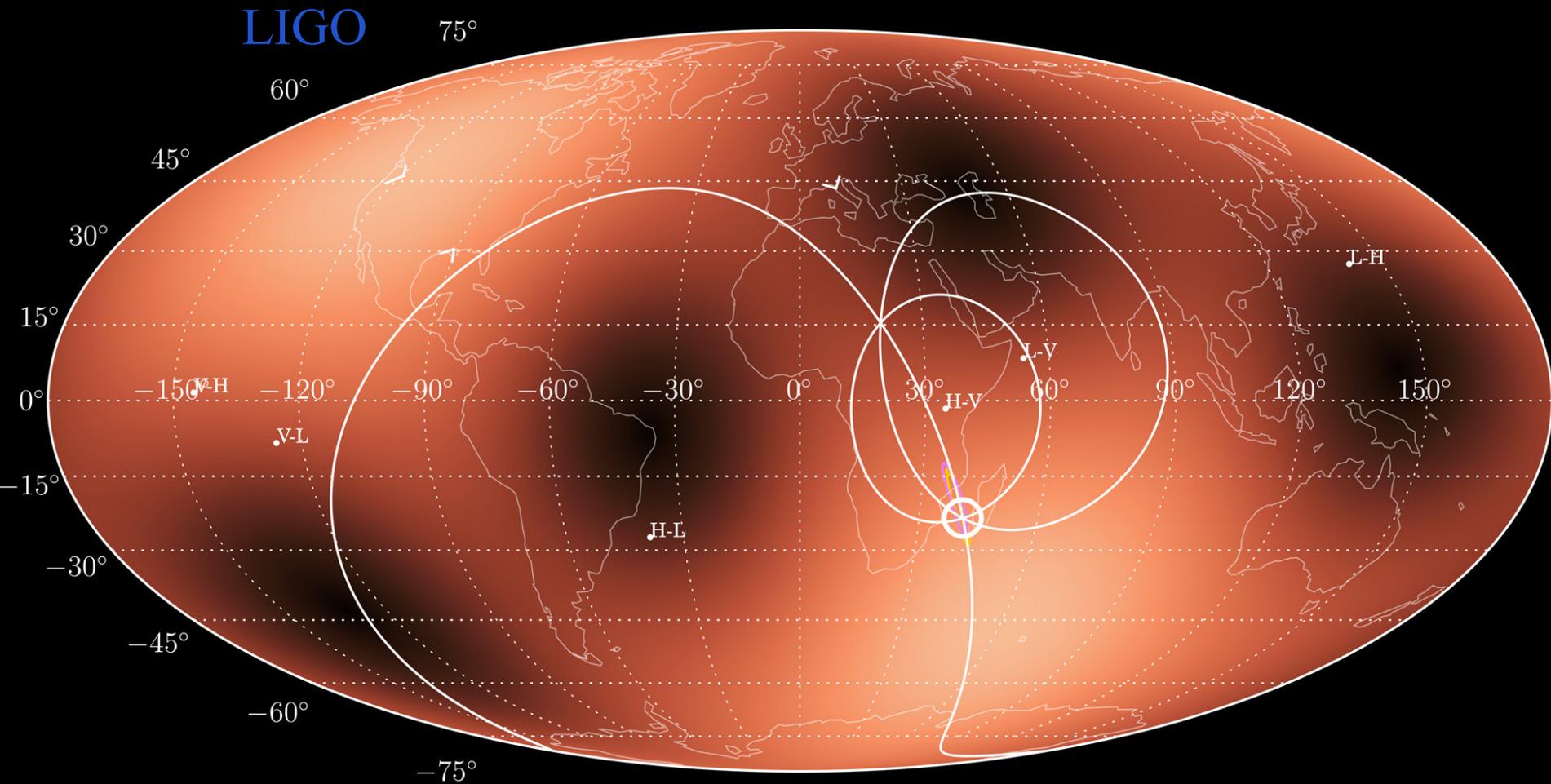
$$\tilde{s}(f) = \int_{-\infty}^{\infty} e^{-2\pi ift} s(t) dt.$$

A commonly used quantity for sensitivity curves is the square root of the PSD or the amplitude spectral density

$$\sqrt{S_s(f)} = h_n(f) f^{-1/2}$$

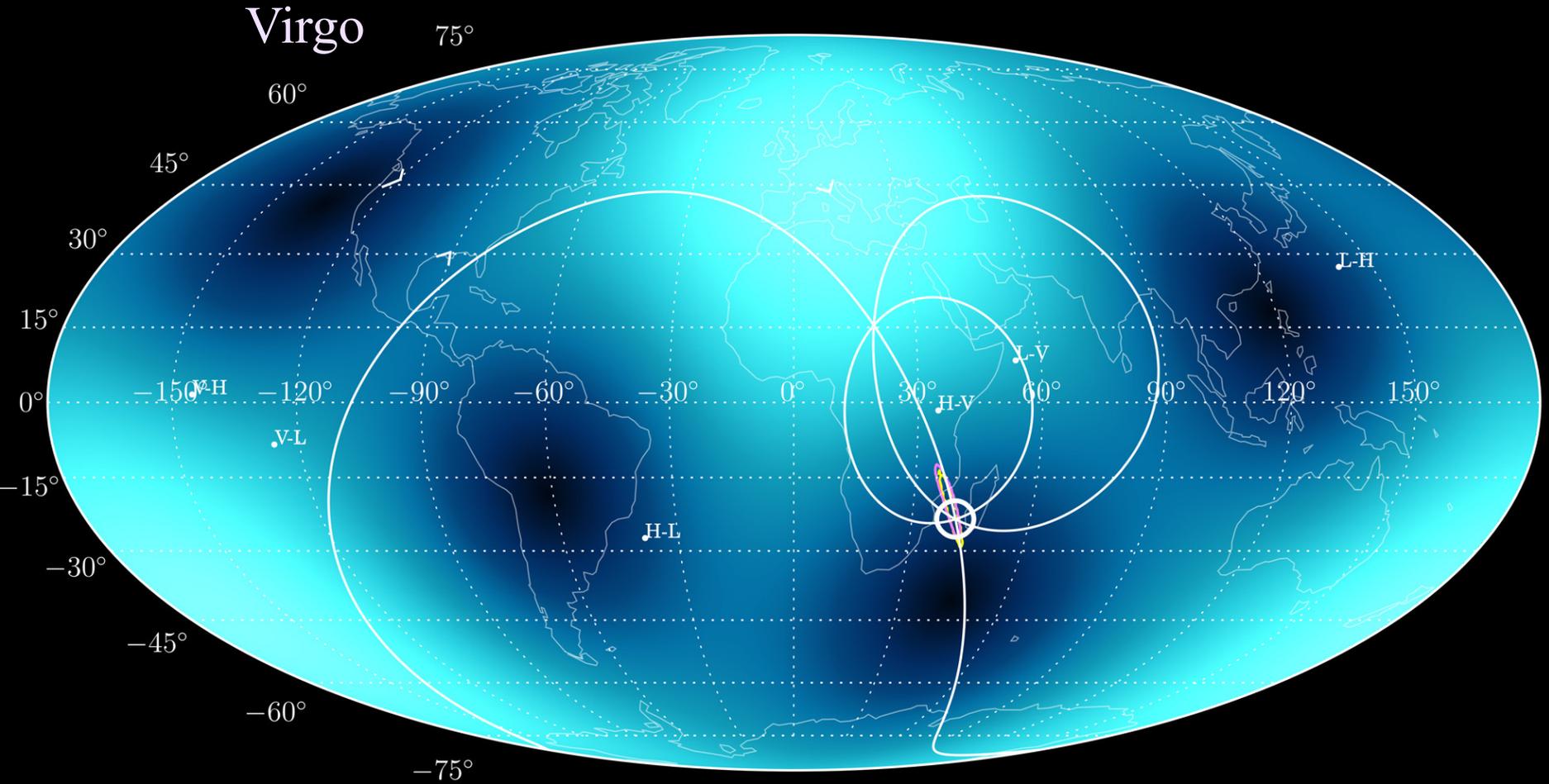


LIGO Antenna Patterns





Virgo Antenna Patterns



The Gravitational-Wave Spectrum

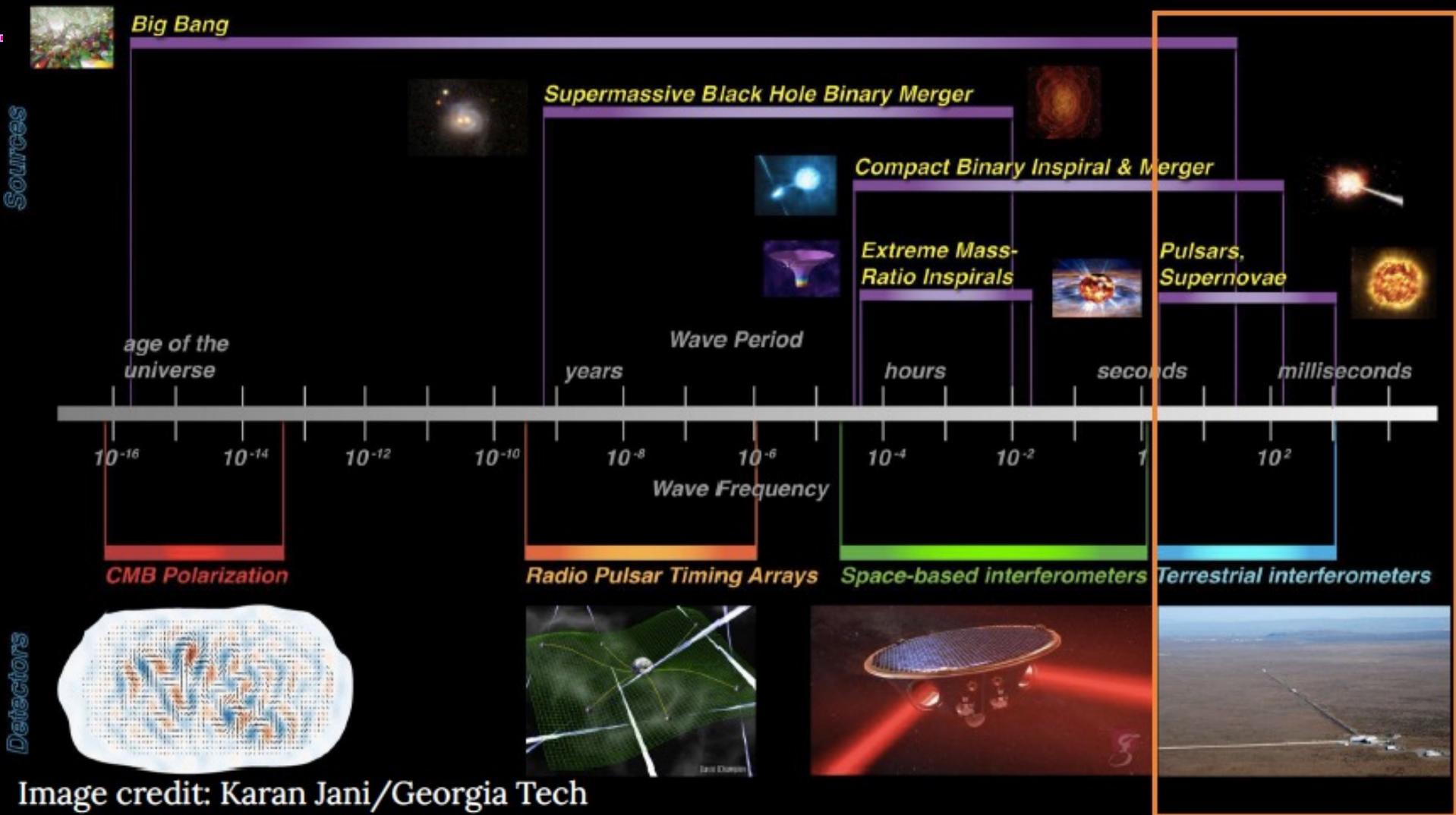
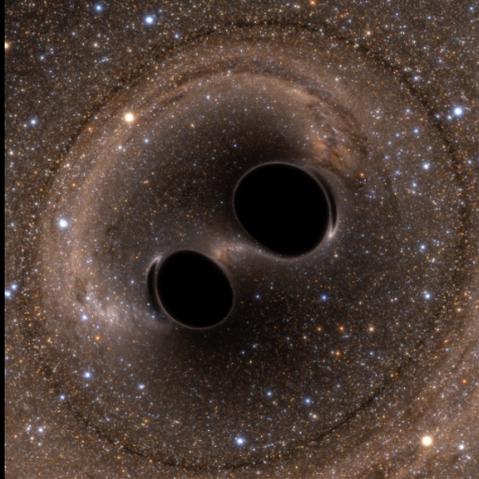


Image credit: Karan Jani/Georgia Tech

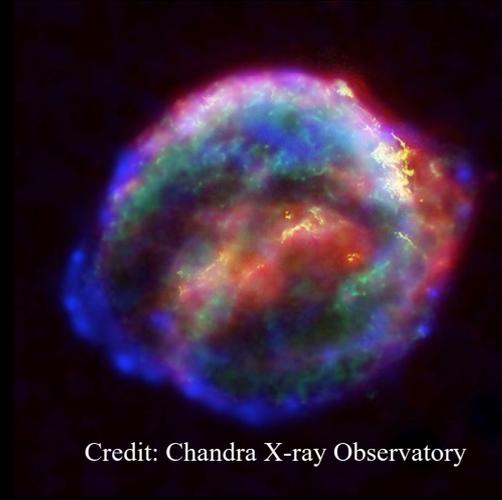
The Astrophysical Gravitational-Wave Source Catalog



Coalescing Binary Systems

- Black hole – black hole
- Black hole – neutron star
- Neutron star – neutron star
- modeled waveform

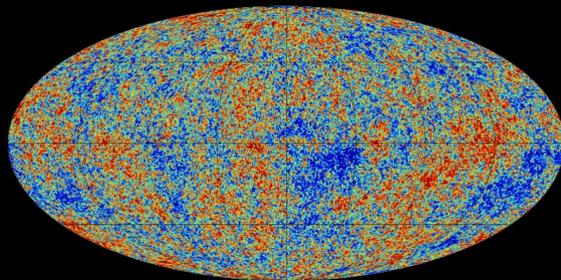
Credit: Bohn, Hébert, Throwe, SXS



Transient 'Burst' Sources

- asymmetric core collapse supernovae
- cosmic strings
- ???
- Unmodeled waveform

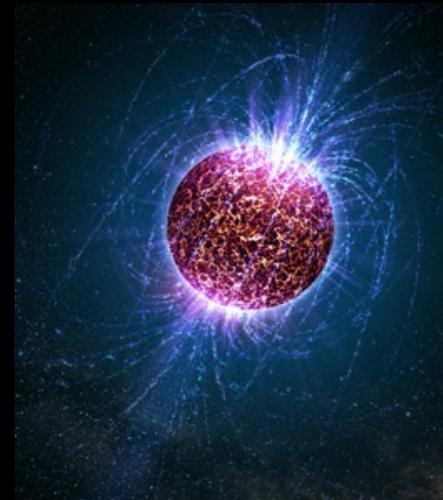
Credit: Chandra X-ray Observatory



Cosmic GW Background

- residue of the Big Bang
- probes back to $< 10^{-15}$ s
- stochastic, incoherent background
- Difficult (impossible?) for LIGO-Virgo to detect

Credit: Planck Collaboration



Continuous Sources

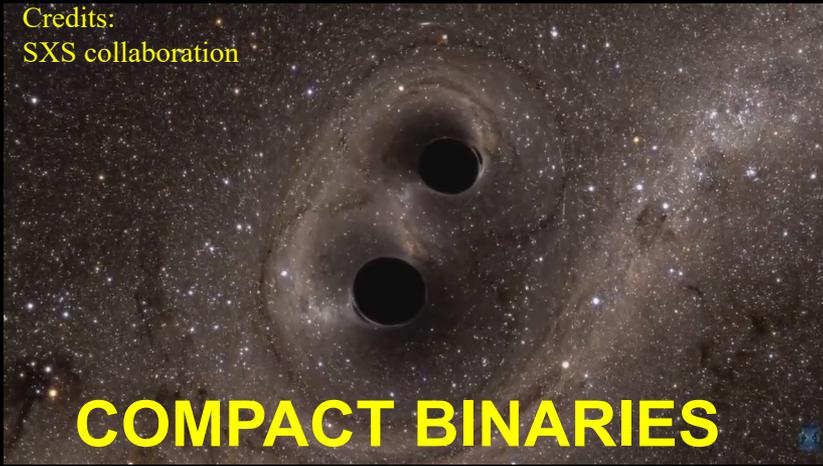
- Spinning neutron stars
- monotone waveform

Credit: Casey Reed, Penn State

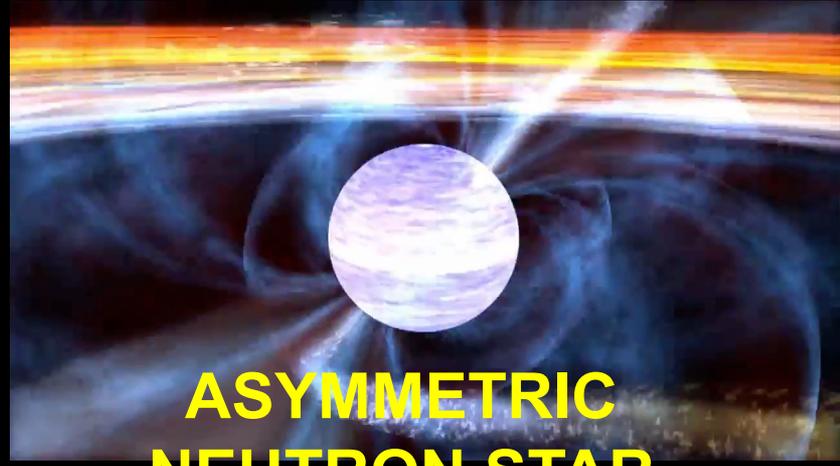
Gravitational Wave Targets

TRANSIENT

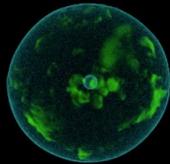
PERSISTENT



**COMPACT BINARIES
(CBC)**



**ASYMMETRIC
NEUTRON STAR**



**BURSTS: Core collapse
Supernovae**



**STOCHASTIC
BACKGROUND**

**MATCHED
FILTER**

UNMODELED



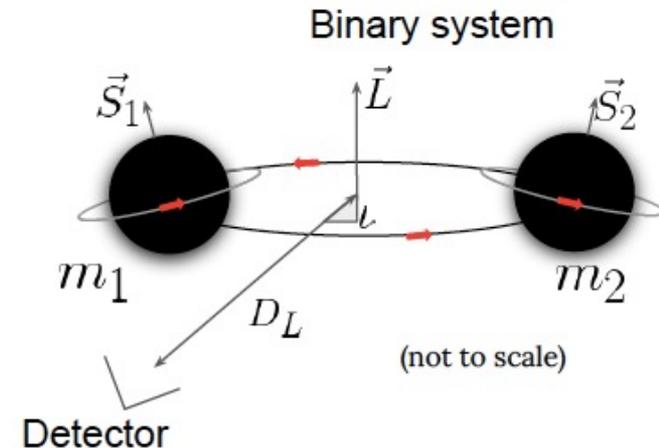
The Matched Filter is the best linear approach to extract a signal of known shape when it is embedded in a stationary Gaussian noise

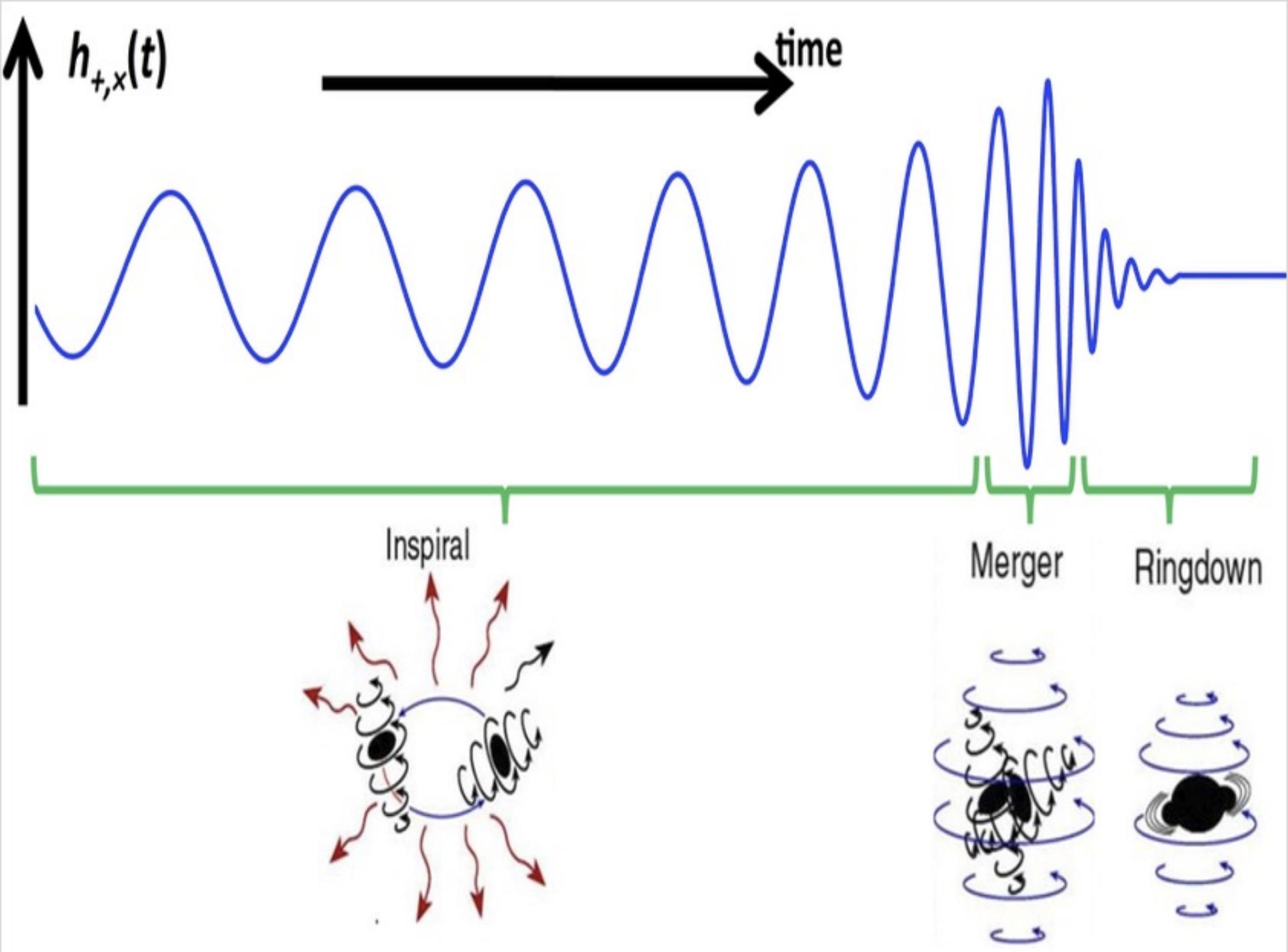
Modelling colliding black holes

What will the signals from these systems look like in the data?

The signal from a binary system made up of black holes will be described by fifteen parameters

- Intrinsic parameters:
 - Component Masses: $m_1 m_2$
 - Component spins in each direction: $s_{1x} s_{1y} s_{1z} s_{2x} s_{2y} s_{2z}$
- Extrinsic Parameters:
 - Location: Right Ascension and Declination
 - Inclination angle between line of sight and orbital plane, i
 - Polarisation angle,
 - Phase at coalescence
 - Luminosity distance, D_L
 - Time of coalescence





Extracting Astrophysical Parameters from GW Waveforms

- Compact object parameters encoded in the waveforms:

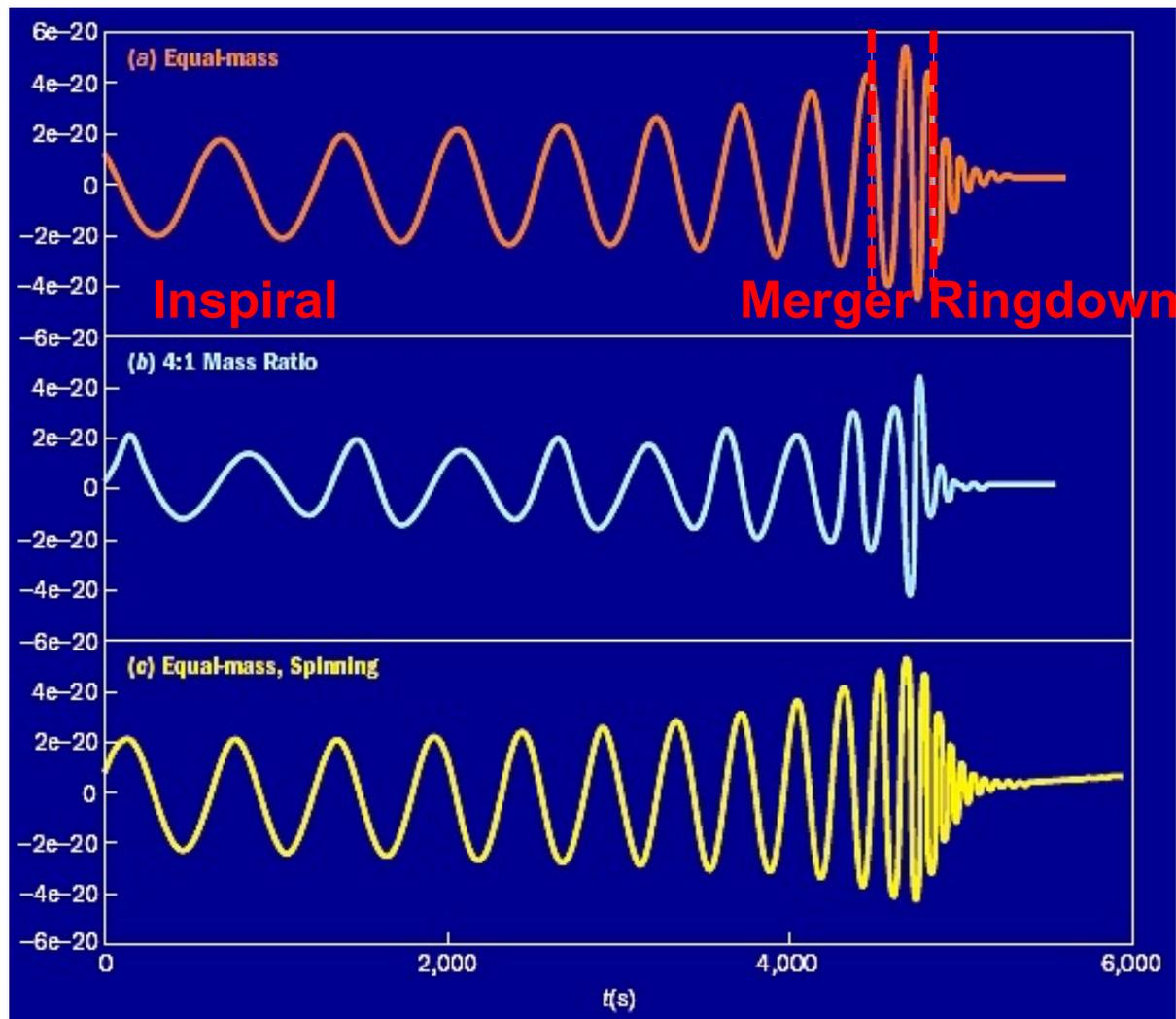
- » Constituent masses, constituent spins, sky location, luminosity distance, orbital inclination, time of arrival

- Intrinsic degeneracies make parameter estimation difficult!

- » E.g., luminosity distance vs. inclination angle

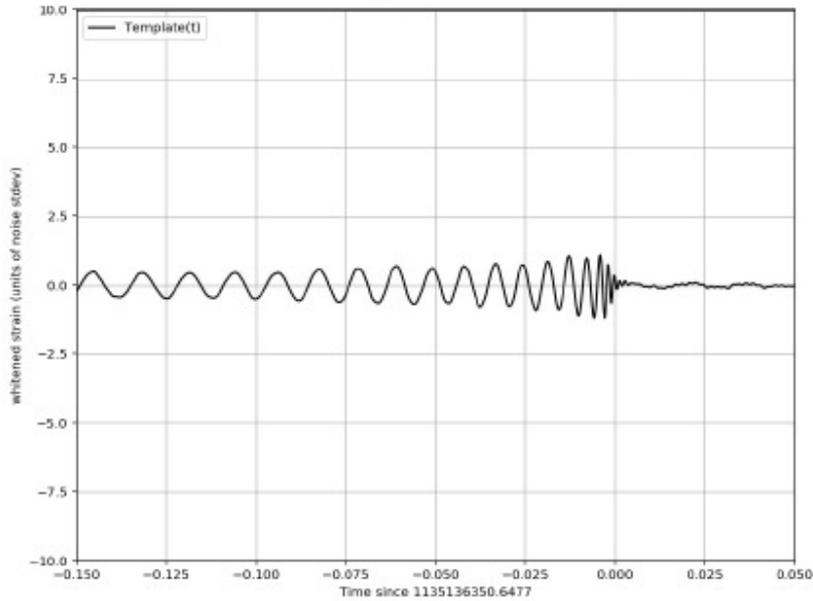
- The SNR of the waveform matters

- » often buried in detector noise; lower SNR obscures parameter estimation

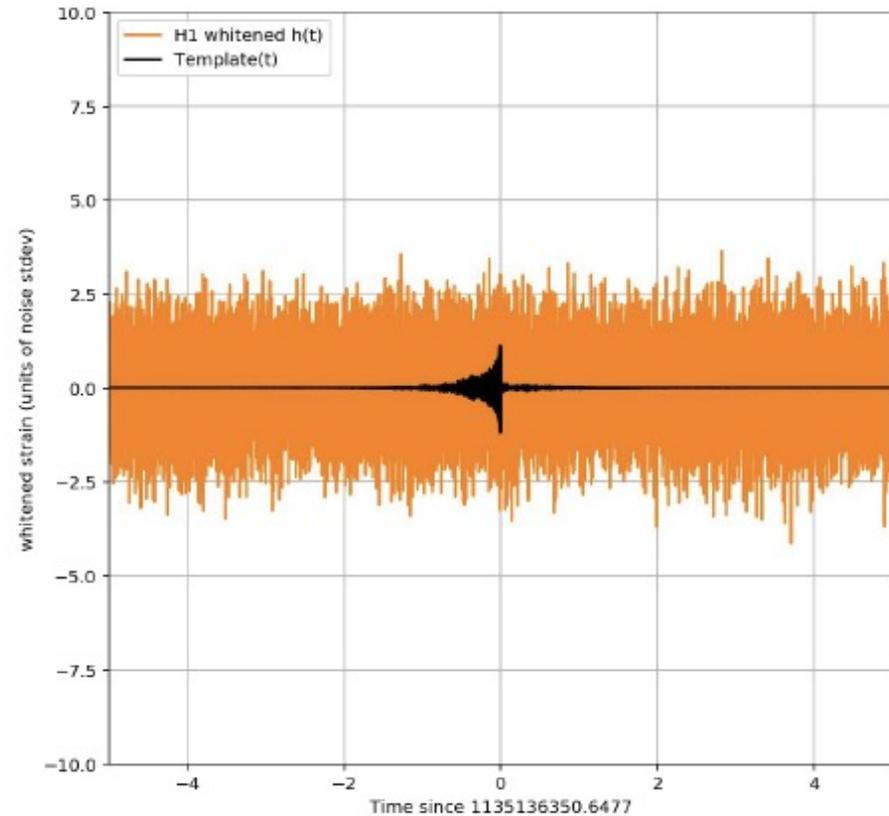


Detection problem

We know what the signal looks like



But it is buried in detector noise



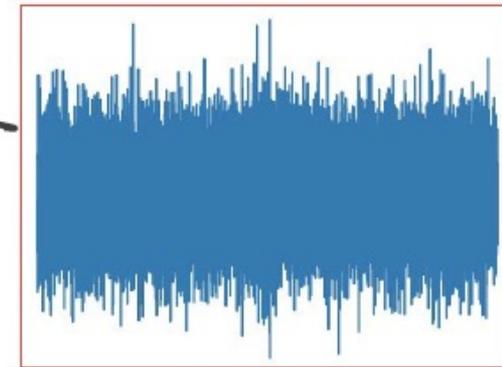
Matched filter

Optimal for signals:

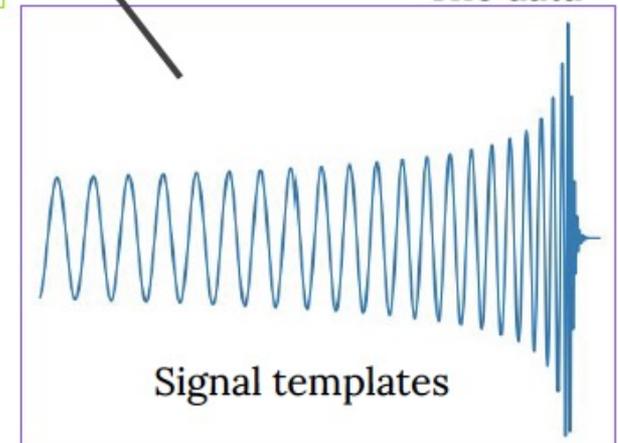
- in stationary Gaussian noise
- with known PSD

(Wainstein and Zubakov, 1962)

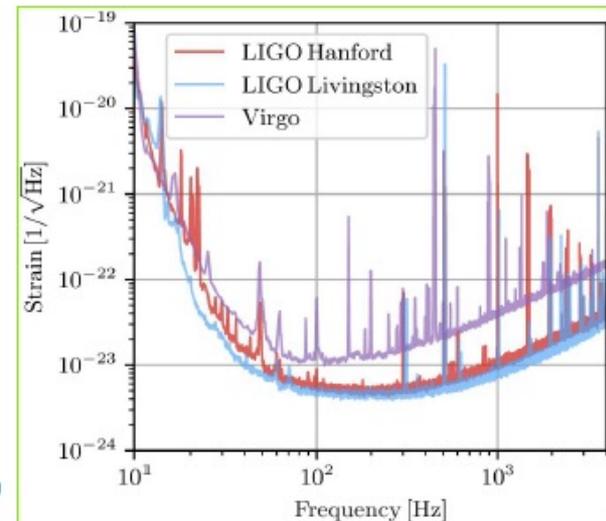
$$(s|h) = 4\Re \int_0^\infty \frac{s(f)\tilde{h}^*(f)}{S_h(f)} df$$



The data

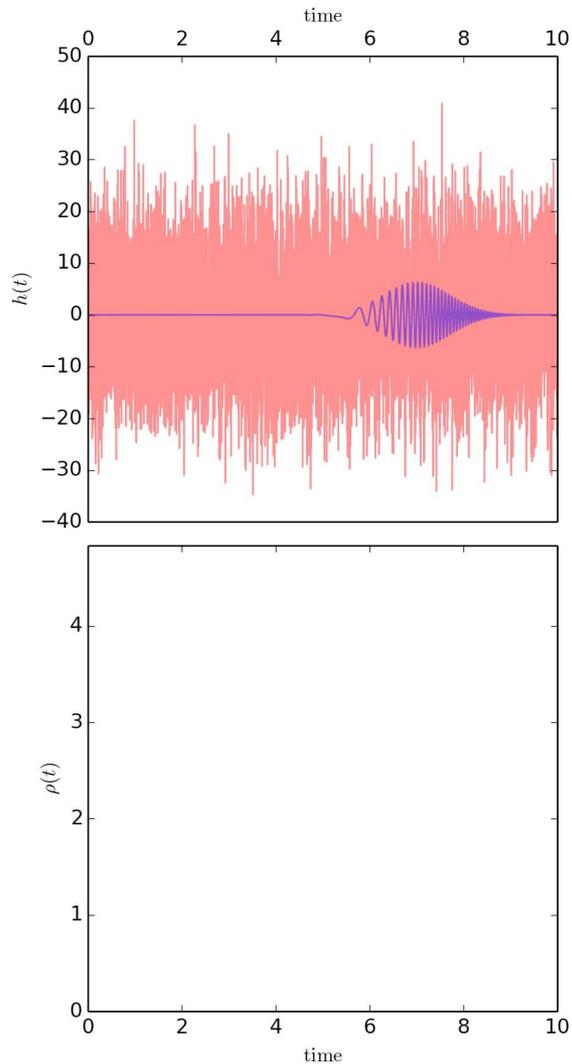


Signal templates



PSD

Assessing Statistical Significance: Modeled Search

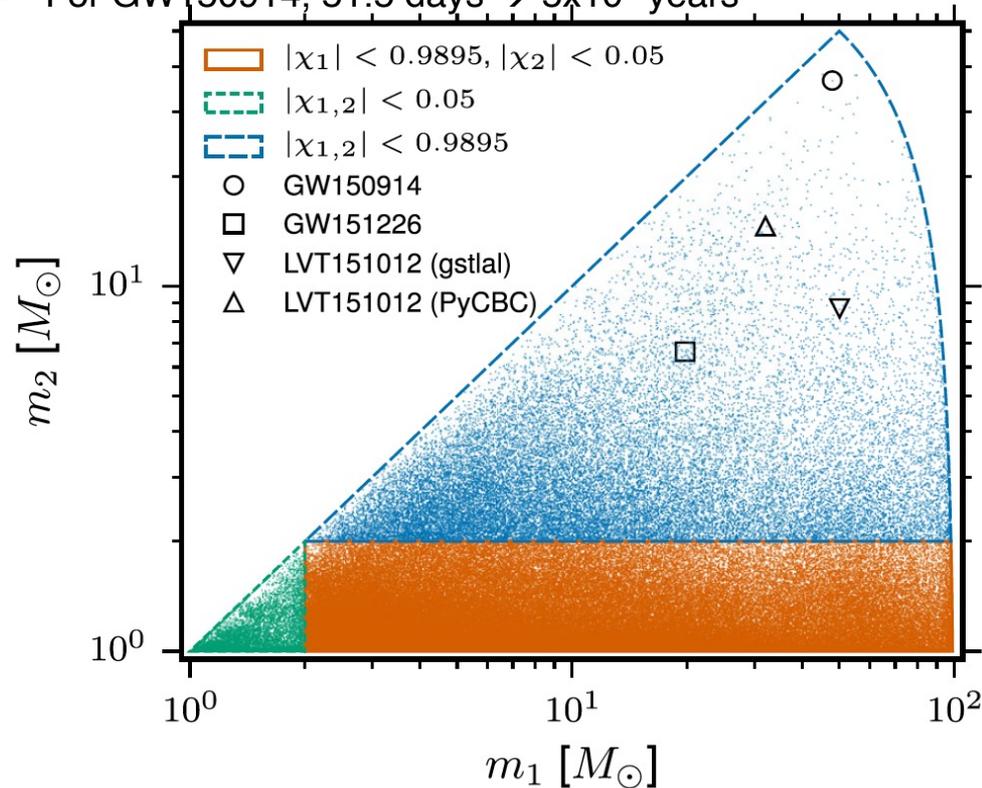


Simulation: Reed Essick, LIGO MIT

- Matched filter search: X-correlation of L1, H1 data streams

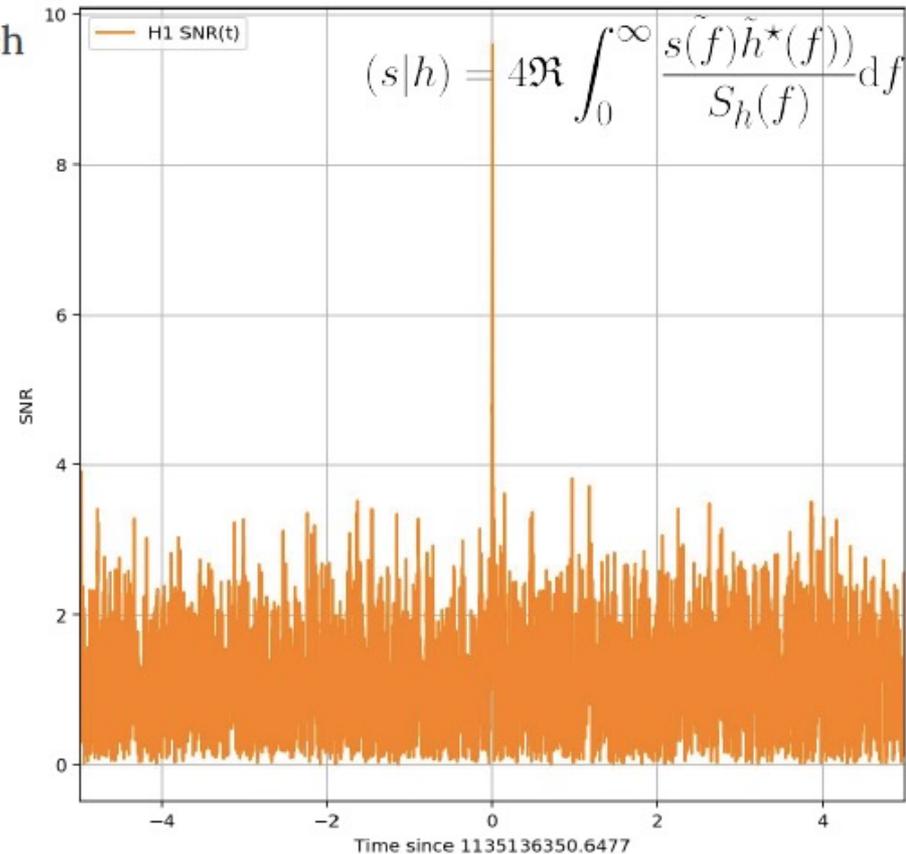
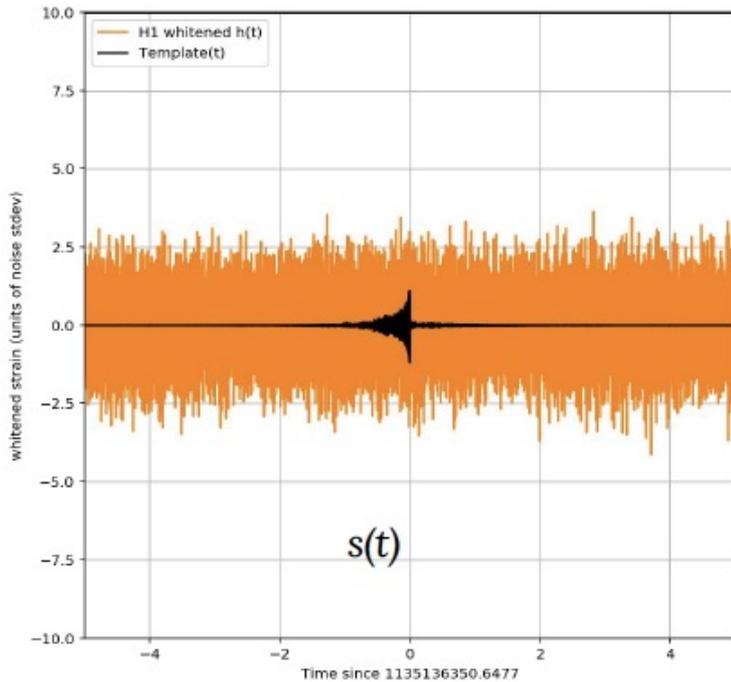
$$\rho = \frac{\langle s|h \rangle}{\sqrt{\langle h|h \rangle}} \quad \langle a|b \rangle = 4\text{Re} \int_{f_{\text{low}}}^{f_{\text{high}}} \frac{\tilde{a}(f)\tilde{b}(f)}{S_n(f)} df$$

- Background computed from time-shifting coincident data in 100 ms steps
 - For GW150914, 51.5 days $\rightarrow 5 \times 10^6$ years



SNR time series

We end up obtaining a time series of SNR values for each template. The peaks in this time series are triggers



Modelling the Waveforms

Matched filtering relies on knowing the shape of the signal.

For CBC waveforms we can model the signals with **template waveforms**.

We construct **template banks** of waveforms that vary over the intrinsic parameters.

4D template banks - $\{m_1, m_2, s_{1z}, s_{2z}\}$

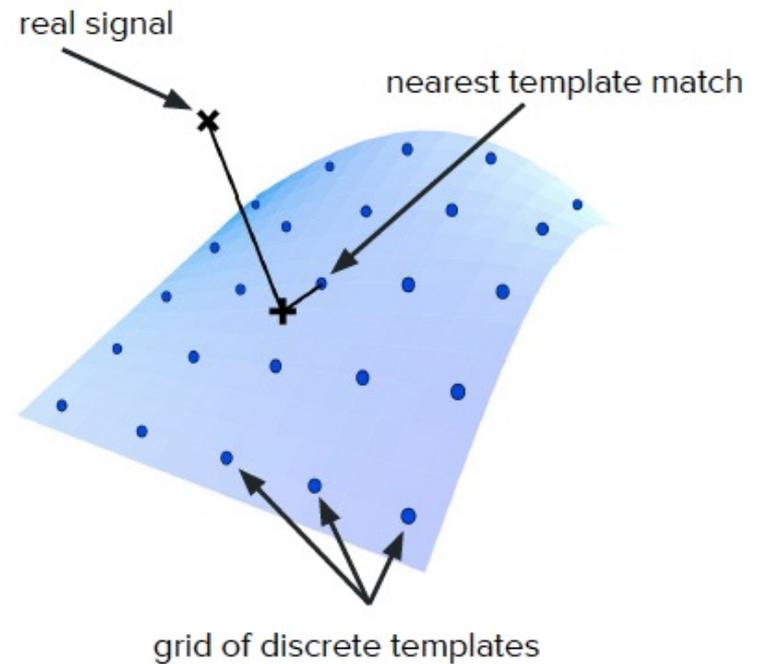


image credit: arXiv:gr-qc/9511032

How many templates do we need?

If the signal perfectly matches the template we will have an optimal SNR.

Any **mismatch causes an SNR loss.**

Construct banks with a dense grid of templates such that any signal will be close enough to the nearest template.

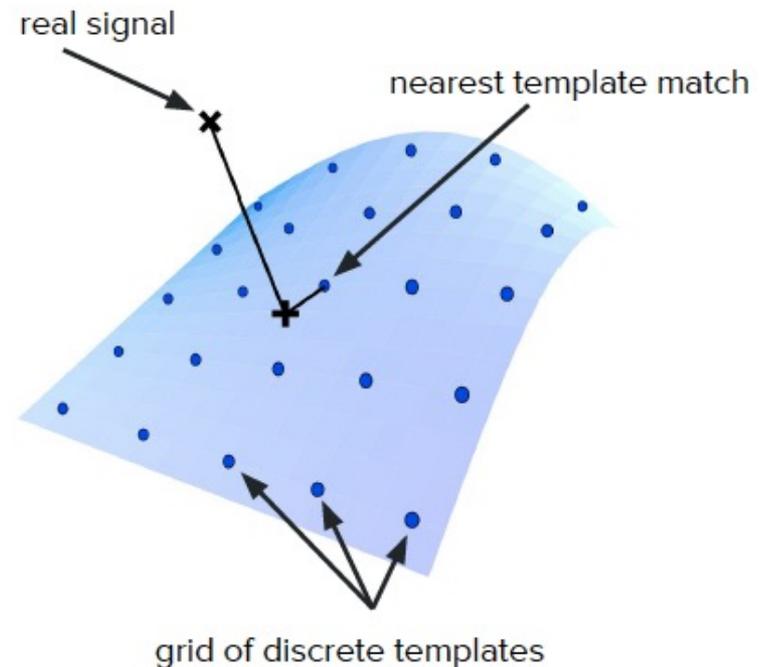


image credit: [arXiv:gr-qc/9511032](https://arxiv.org/abs/gr-qc/9511032)

$$N_{\text{templates}} \sim \mathcal{O}(10^5 - 10^6)$$

Which waveforms do we use?

As mentioned earlier, the parameters with most impact on the signal waveform are the masses and aligned spins of the components

We place templates within the bank randomly, but only if the match ($h|h$) between templates is below a specific threshold.

This means that we end up with a bank which should match well to any signal within this parameter space

The template on the right has been used for the PyCBC-Broad search for many recent publications, and contains ~400k templates

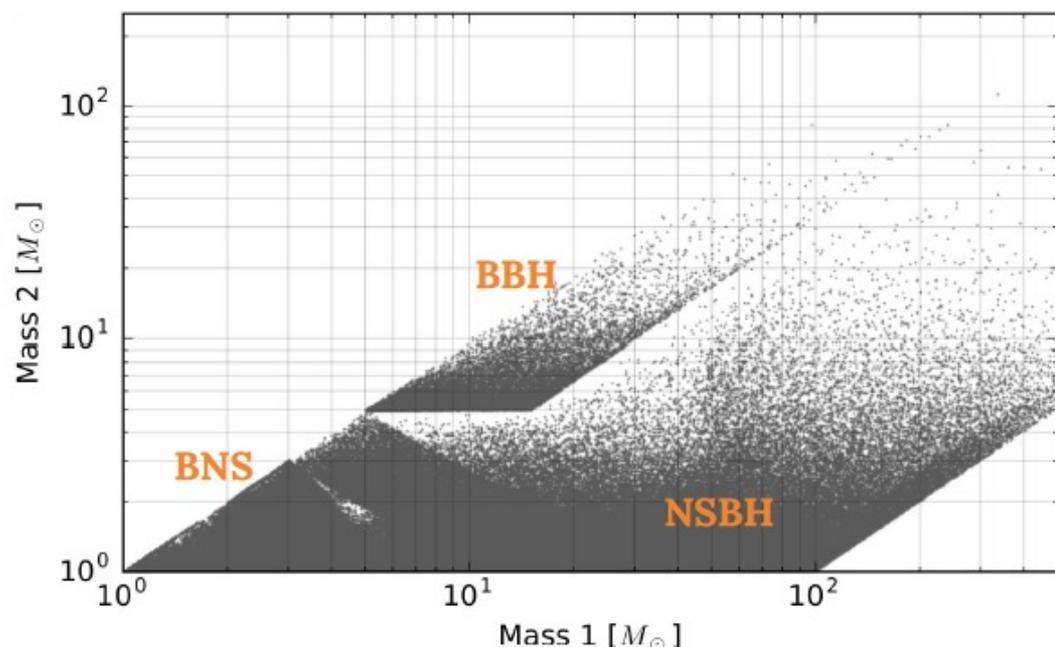


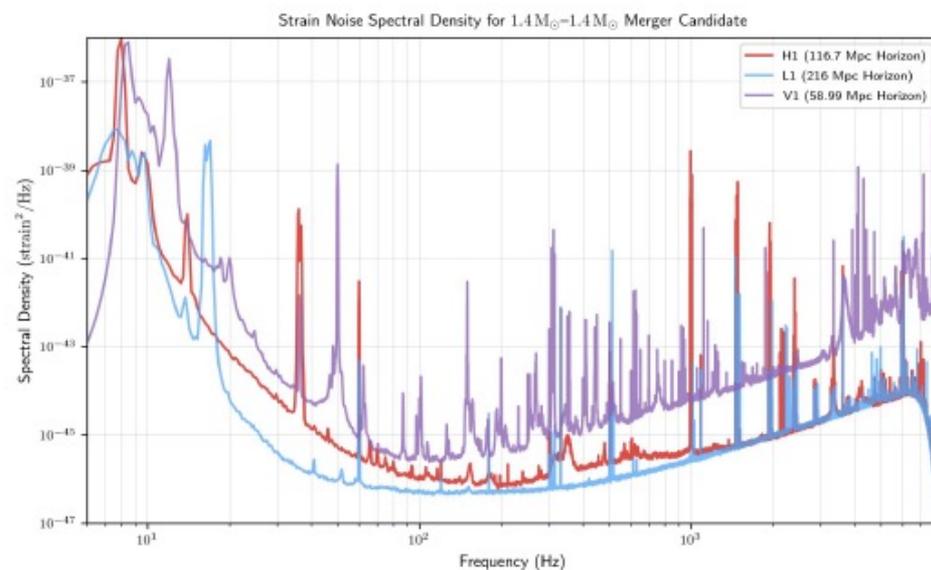
Image credit: Dal Canton and Harry (2017)

Revisiting our assumptions

In reality, LIGO data is not well-modeled.

- **non-stationary** over short and long time-scales
- **non-Gaussian**

The only characterization of the LIGO noise is from observations.



LIGO and Virgo PSD for O3.

The PSD shows a **measure of the sensitivity** and how the **noise varies over frequency** bins.

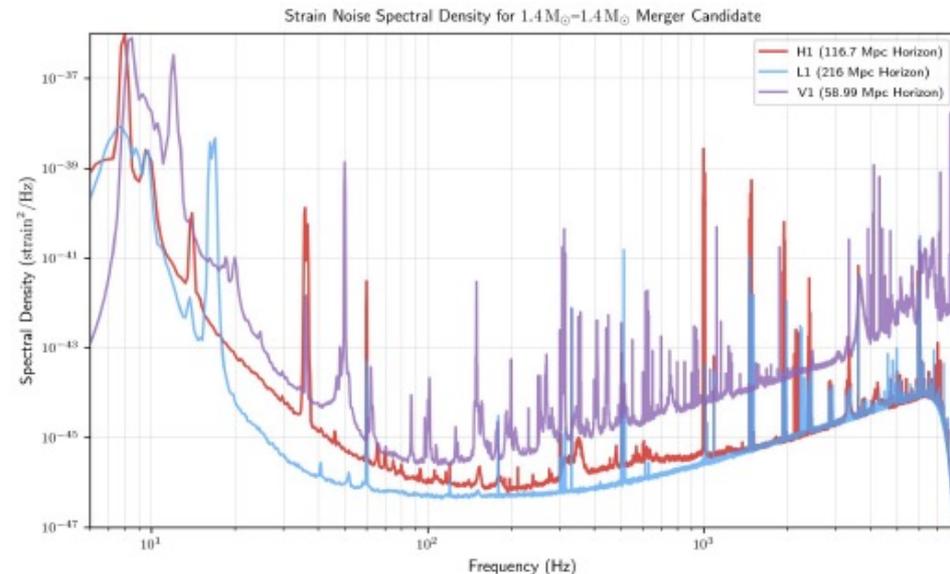
Revisiting our assumptions – colored noise

LIGO noise is not **white** - the power is not distributed evenly across frequencies.

$$\rho(t) = \int_{-\infty}^{+\infty} d\tau \hat{h}(\tau) \hat{d}(t + \tau)$$

whitened template
and data

$$\hat{d}(\tau) = \int_{-\infty}^{+\infty} df \frac{\tilde{d}(f)}{\sqrt{S_n(|f|)}} \exp 2\pi i f \tau$$



LIGO and Virgo PSD for O3.

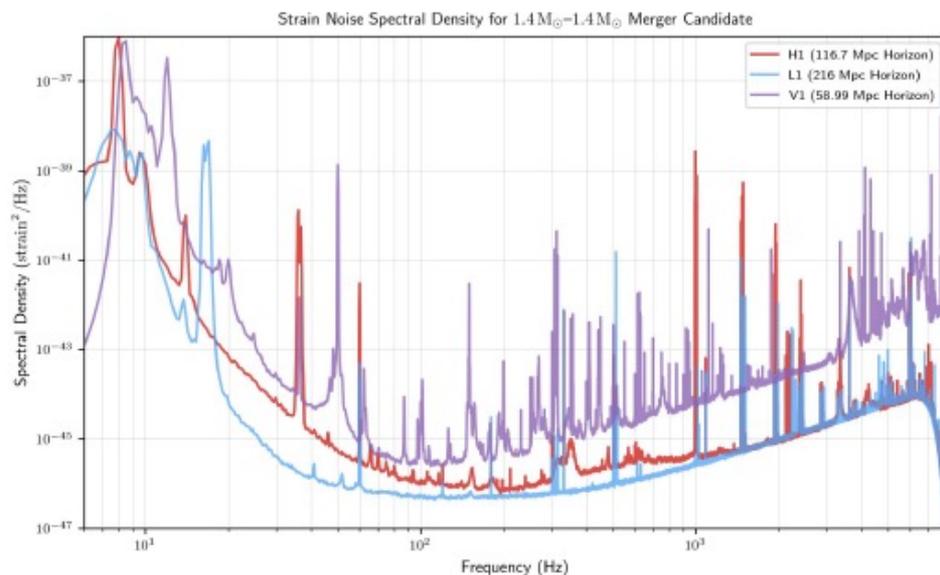
The **PSD varies across frequency** bins, so the data needs to be whitened before filtering, essentially scaled by the PSD in frequency space.



Revisiting our assumptions – non-stationarity

Noise properties can vary over **long timescales**.

We must continuously track the noise properties and update our estimate of the PSD.



LIGO and Virgo PSD for O3.

This is a **snapshot of the PSD** for about 1 hour of O3 data, but at another time the PSD could be different.

Non-stationarity

The detector sensitivity is not constant, this can happen rapidly or slowly.

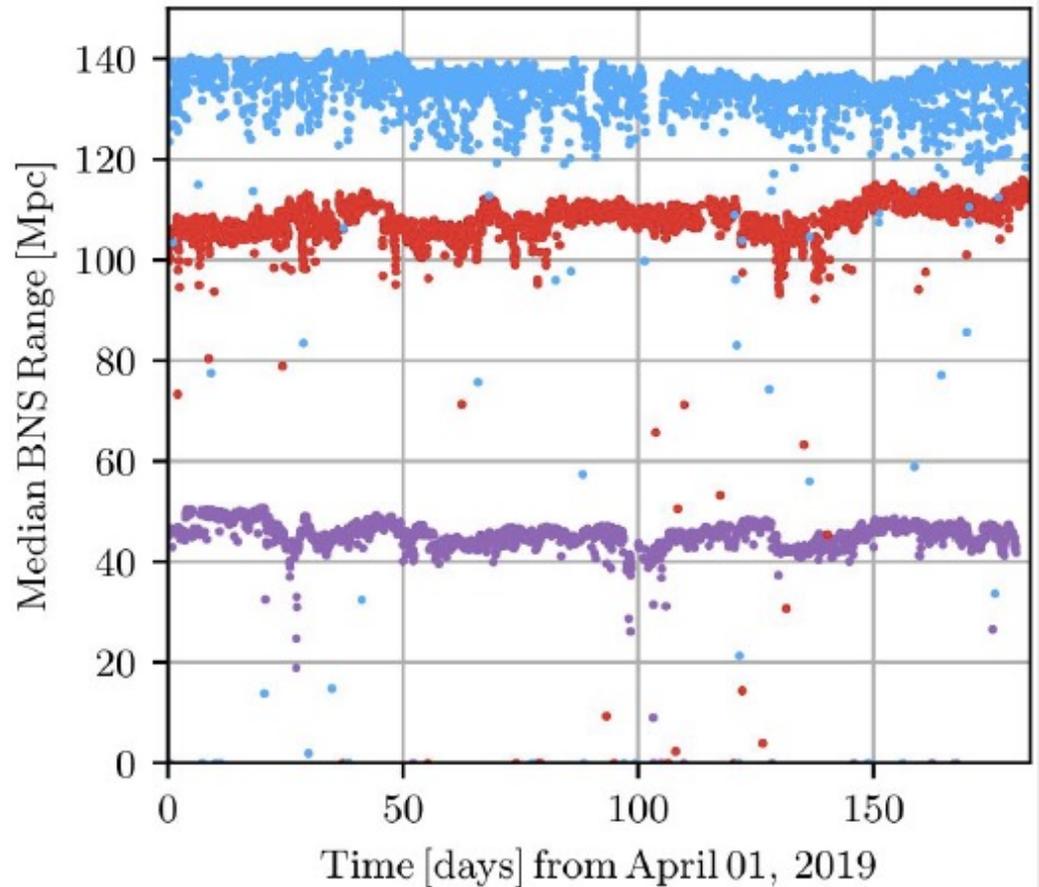


Image from Abbott et al (2020) GWTC-2 2010.14527

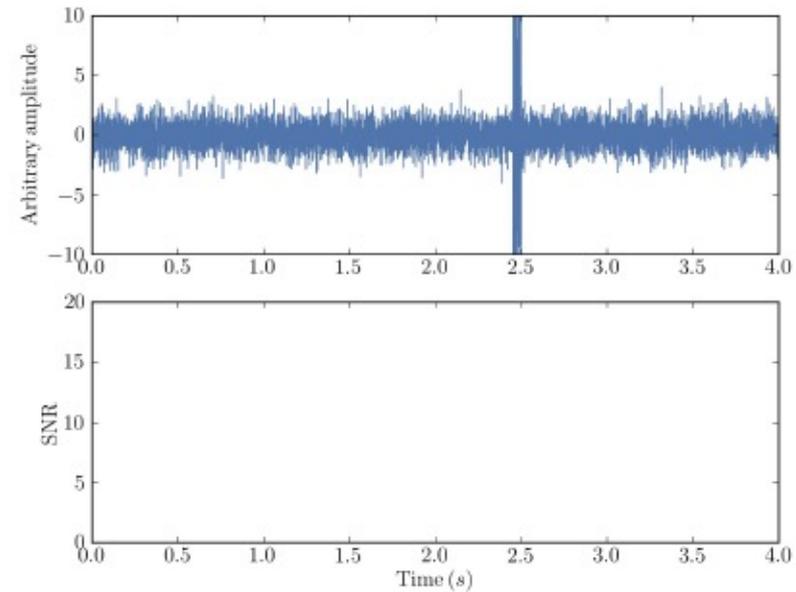


Revisiting our assumptions – non-stationarity

Noise properties can vary over **short timescales**.

Short duration non-Gaussian artefacts in the data are **glitches**.

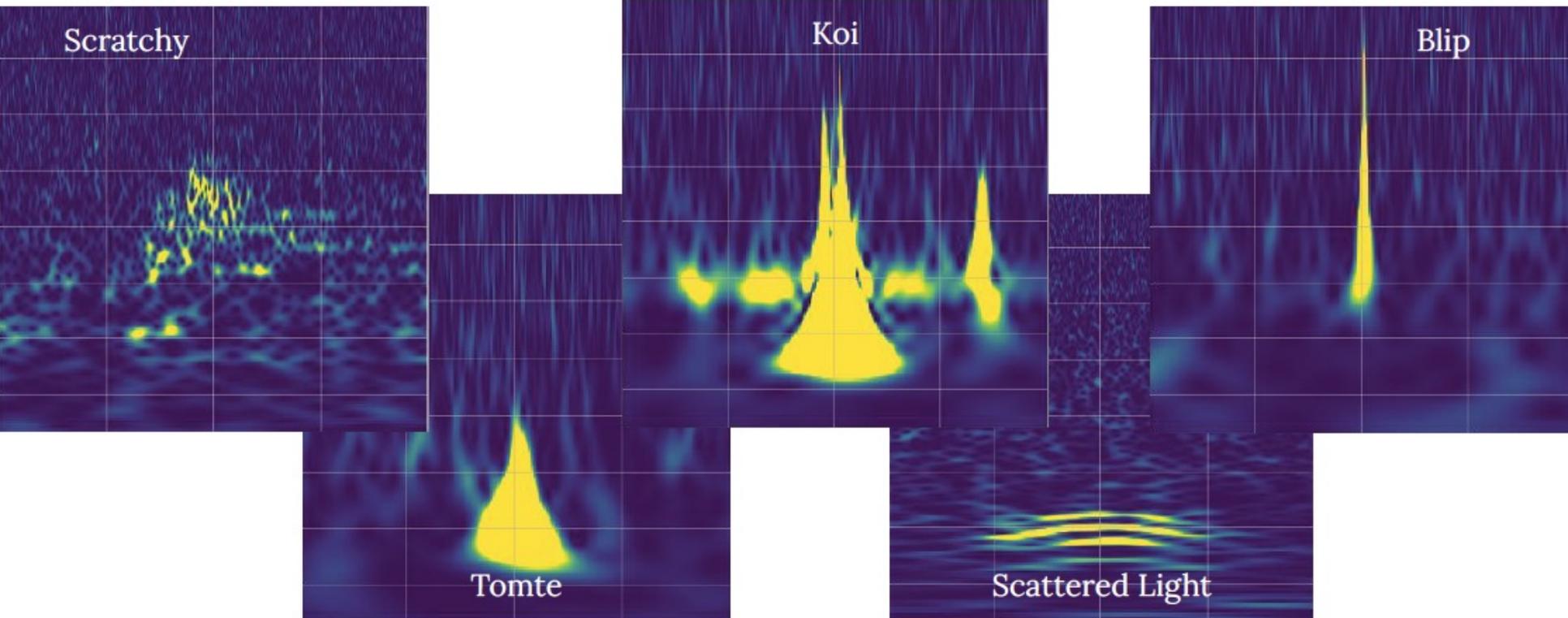
Glitches can be a major problem for matched filtering searches!



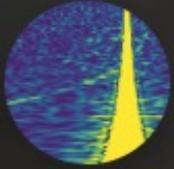
Example of a glitch in LIGO data ringing up an SNR peak in the matched filtering output.
Image credit: Ryan Magee.

Non-Gaussian glitches

Gravity Spy: <https://www.zooniverse.org/projects/zooniverse/gravity-spy>

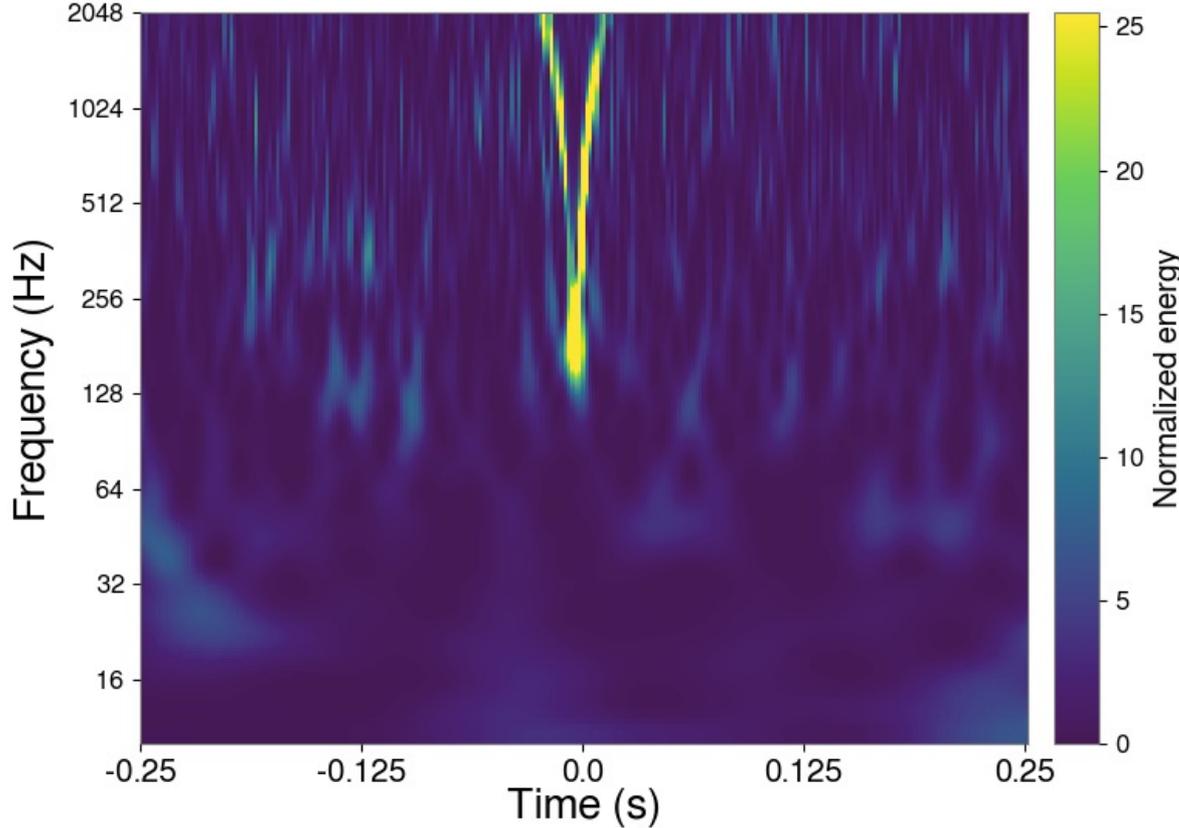


Glitch classification



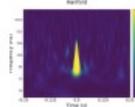
Gravity Spy

Hanford - O3

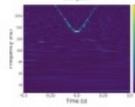


TASK

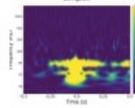
TUTORIAL



Blip



Whistle



None of the Above

Showing 3 of 3 Clear filters

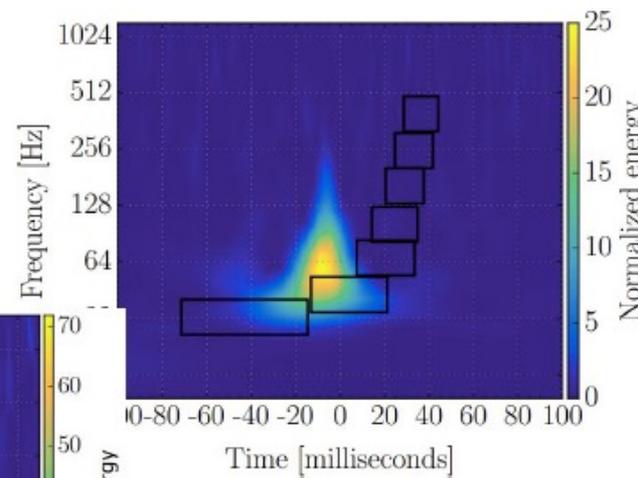
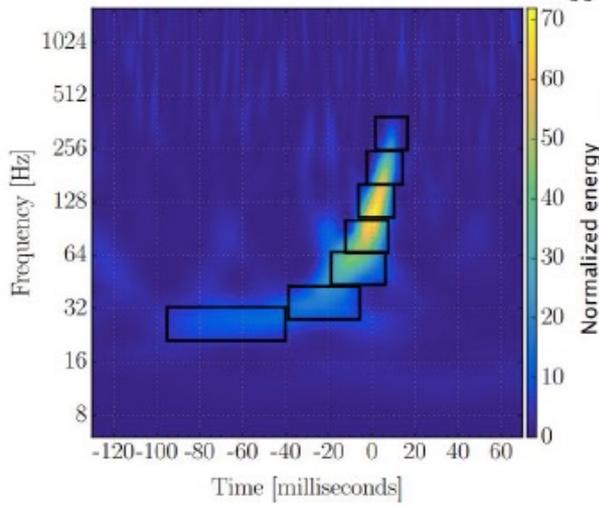
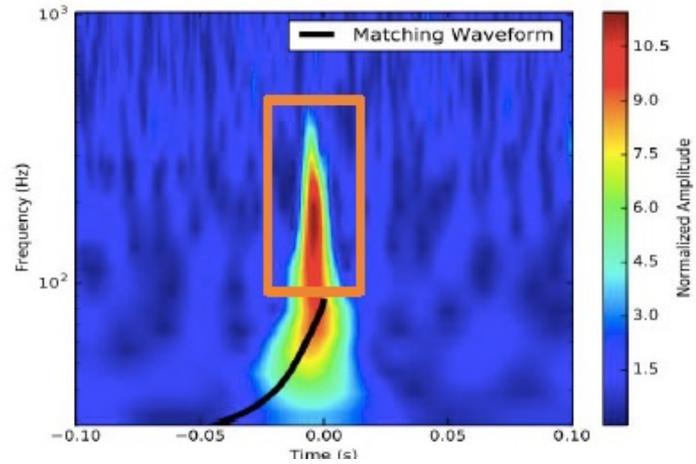
Done & Talk

Done

Beyond Matched Filtering

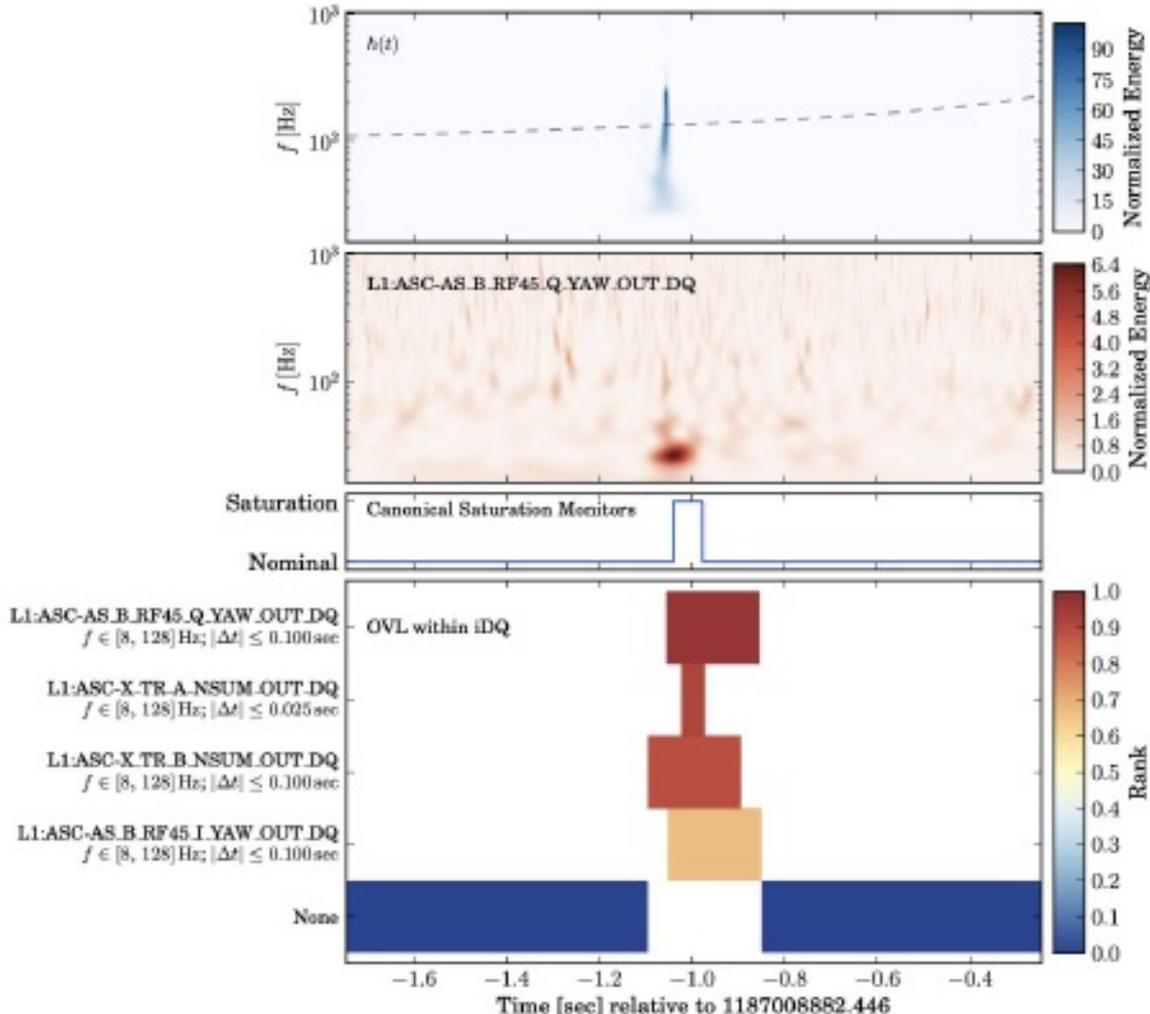
SNR is optimal *if data is Gaussian*. Data is not Gaussian

1. Split into frequency bins and check that the relative amount of power in each bin is correct (right)
2. Check for power above the final frequency of the signal (below)



Images: Nitz 2018 (left), Harry 2019 (above, ODW 1)

Beyond Matched Filtering



Glitch in strain data

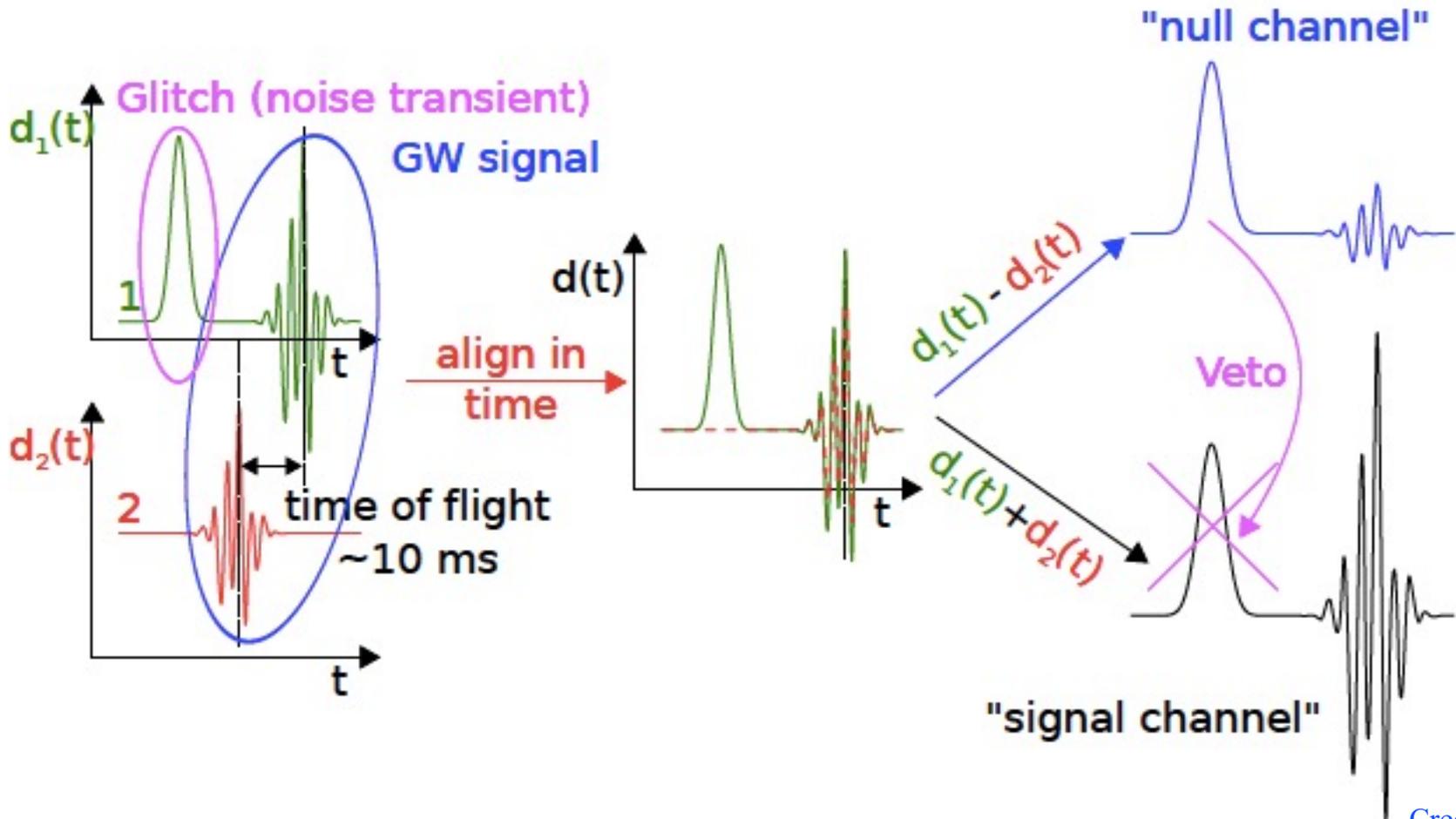
Glitch in auxiliary channel data

Probability of glitch according to several auxiliary channels.

arXiv:2005.12761v1

Coincidence

- Glitches are not correlated between detectors
- GW signals are within light-travel-time between each pair of detectors



Type of searches

Modeled searches

The analyses correlated detector data with template waveforms that model the expected signals



Candidate events that are detected at both observatories with the same template and consistent with the Δt inter-site propagation time are identified



A detection-statistic value ranks likelihood event of being a GW signal
Detection statistic is compared to background to determine the probability that a candidate is due to detector noise

Un-modeled searches

Search for excess power in time-frequency domain (Wavelet, Q-transform, ...)



Combine coherently the excess powers of different detector in a unique data stream
Consider time-delay between detectors
Include antenna pattern factors

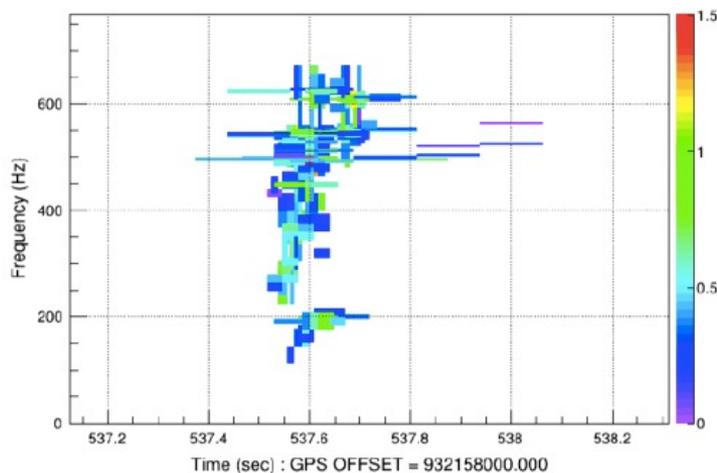


Calculate a detection statistic and compare the one of each candidate to the background distribution

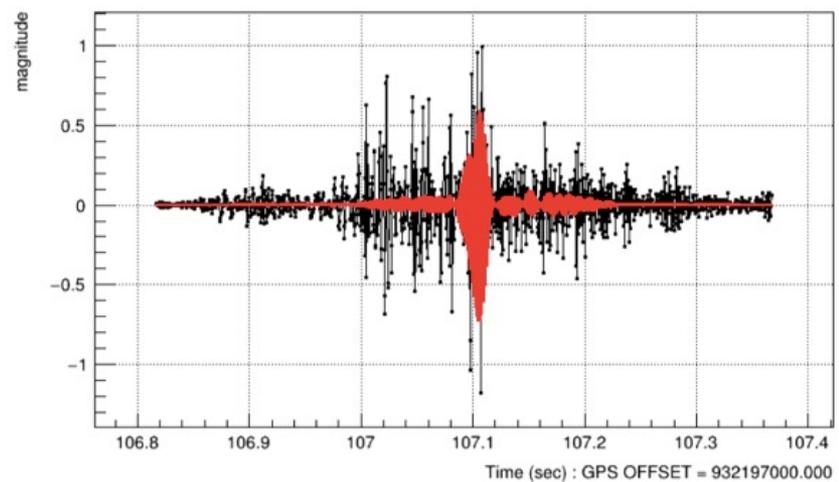
Coherent Waveburst (cWB)

- Coherent Waveburst is an algorithm of Burst search developed at LVC
- Interesting features:
 - » Characterization of signal both in time and frequency (Wavelet)
 - » Coherent analysis (Likelihood approach)
 - » Reconstruction of waveforms and source coordinates
- Waveburst is applied in two steps:
 - » **Production**: production events list
 - » **Post production**: candidate selection

Likelihood 96 - dt(ms) [7.8125:250] - df(hz) [2:64] - npix 186



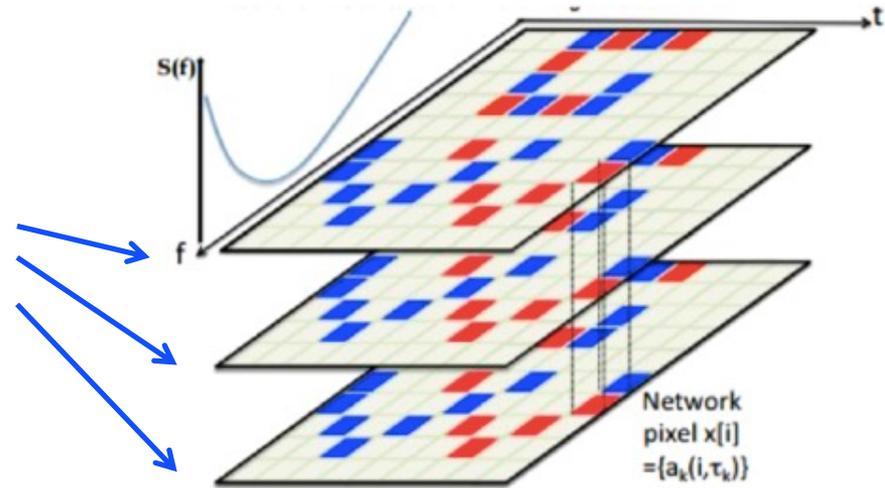
Injected (black) vs reconstructed (red)



Coherent Waveburst

- The pipeline decomposes the data stream, of each detector in the network, at different (dt, df) resolution levels.
- cWB is an Excess power algorithm: minimal assumption on target signal.

Representation of
multilayer
decomposition
of the GW data



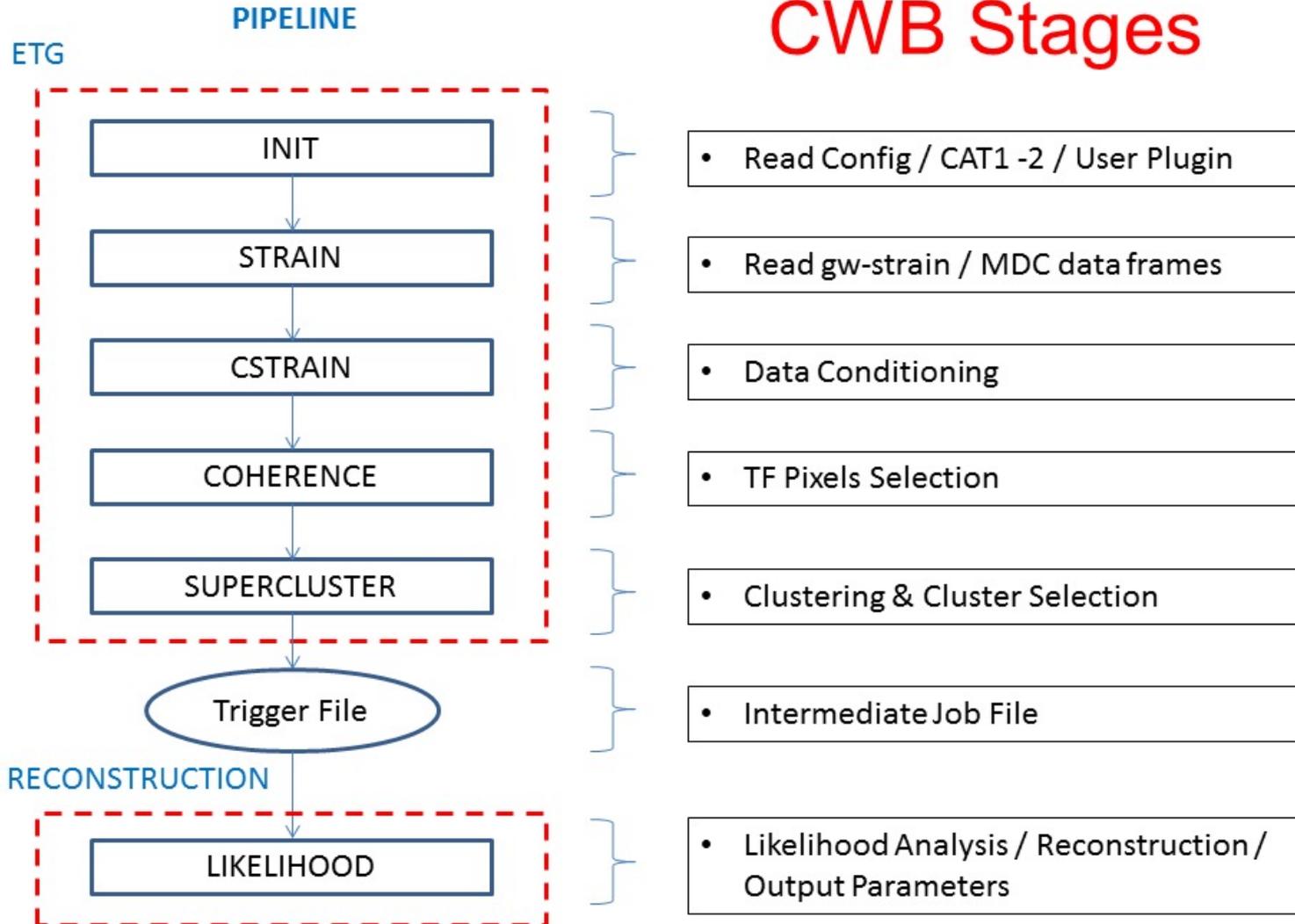
- Time-Frequency decomposition
- Cluster selection, based on black pixel probability
- Constrained Likelihood

Inverse
Wavelet
Transform $\rightarrow h(t)$

$x(t)$ $\xrightarrow{\text{Wavelet Transform}}$

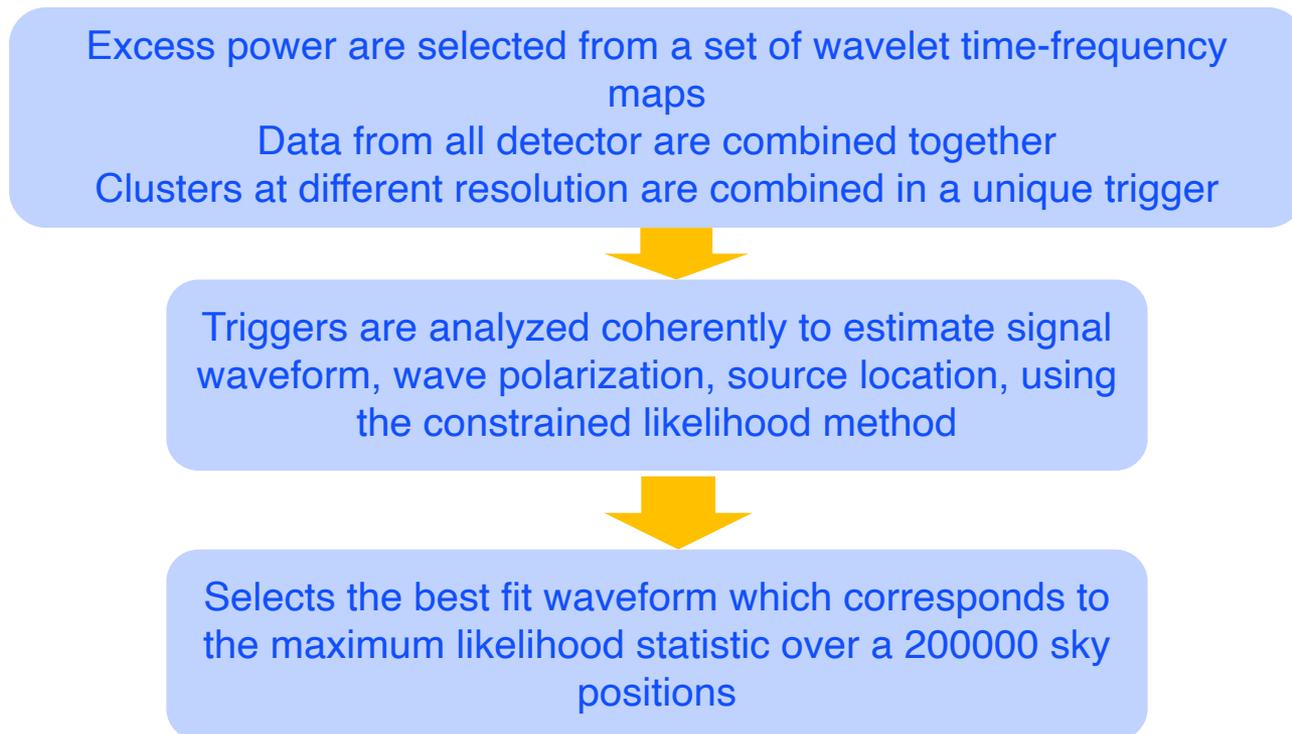
Flowchart

CWB Stages



Flowchart

- Coherent Waveburst is one of the algorithms for **Burst search** developed in LIGO-Virgo
 - [Web page](#)
 - *Phys. Rev. D 93(4), 042004, 2016*

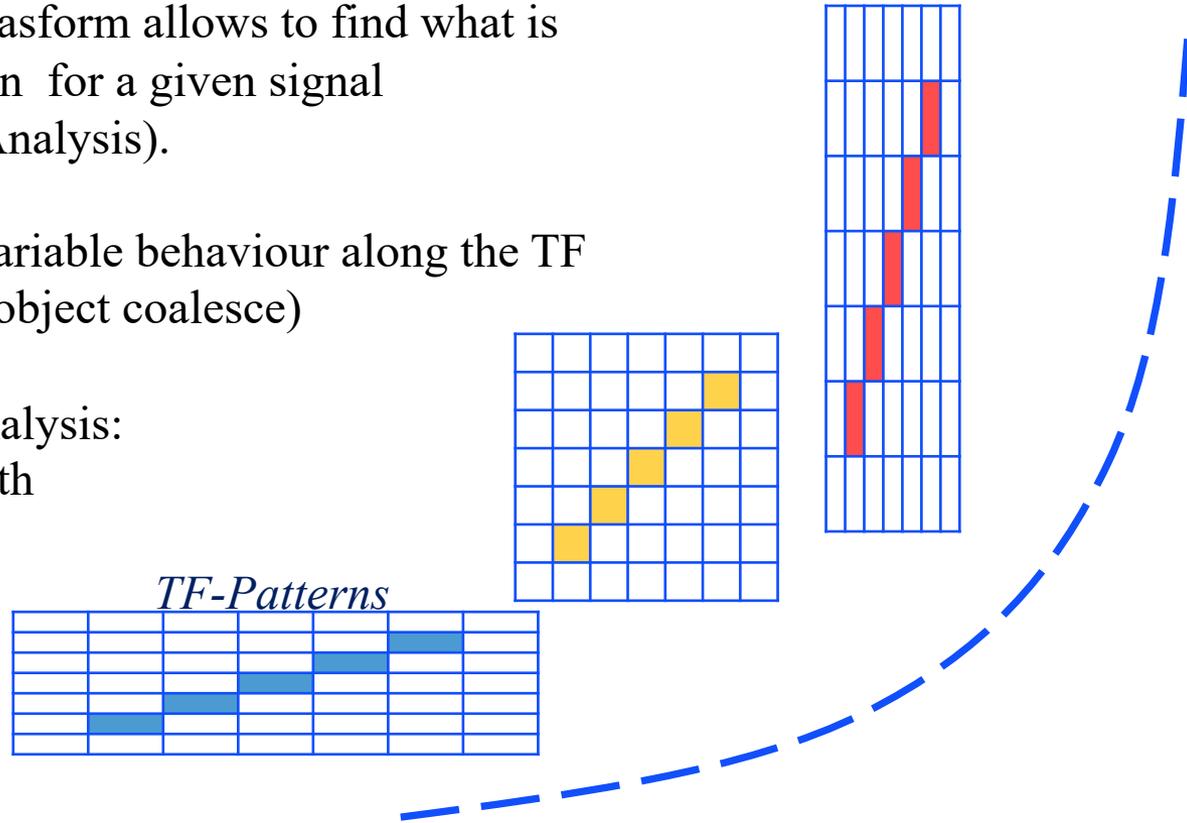


Principal Component Analysis

Using multiple TF transform allows to find what is the optimal resolution for a given signal (Single-Resolution Analysis).

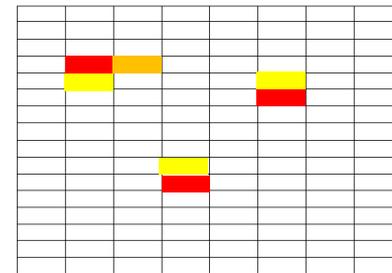
Signal could show variable behaviour along the TF plane (two compact object coalesce)

Multi-Resolution Analysis:
include TF pixels with different resolution

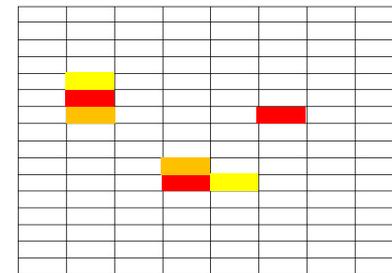


Clustering

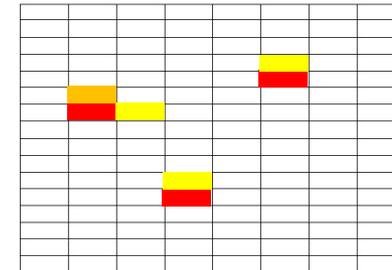
- TF pixels are selected according to coherence between detectors
 - » Coherence verify if the energy of the pixels overcome a threshold
- Coincident TF pixels from different detectors are combined to form a cluster
- The cluster identify an event
- Cluster for each TF map are combined to form a supercluster
- Likelihood is calculated on the supercluster



L1



H1



V1

Likelihood Analysis

- Likelihood Ratio

$$L = \frac{p(x | h)}{p(x | 0)}$$

- Matched filter for bursts

- » **Noise model:** Gaussian Noise

$$p(x | 0) \propto \exp[-x^2 / \sigma^2]$$

- » **Signal model:**

$$p(x | 0) \propto \exp[-(x - \xi)^2 / \sigma^2]$$

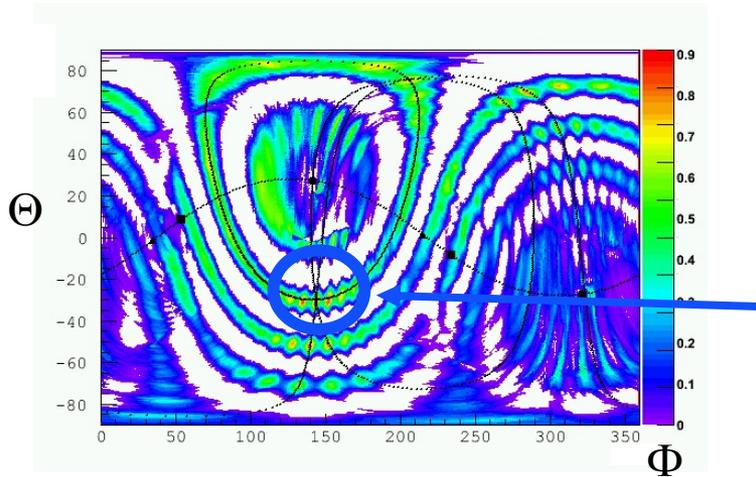
$$\xi(t) = F_+(\theta, \varphi, \psi)h_+(t) + F_\times(\theta, \varphi, \psi)h_\times(t)$$

Detector Noise Variance

Detector Response

- Find best solution of h_+ , h_\times for maximum of L

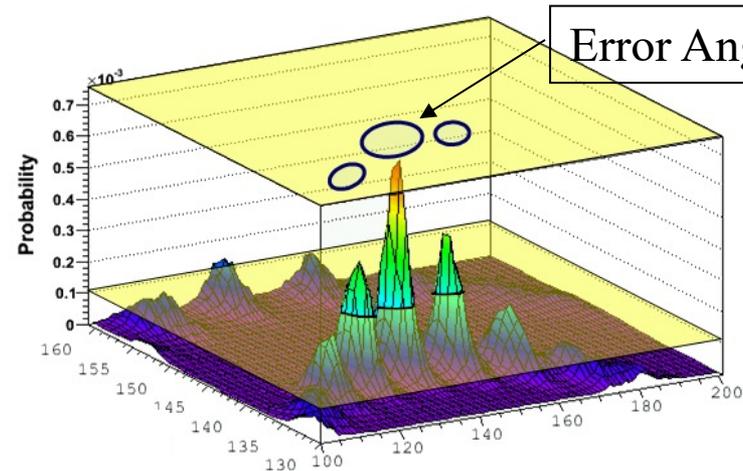
Source localization



Likelihood Sky Map shows how consistent are reconstructed waveforms and time delays as a function of Θ , Φ .

Maximum likelihood point to reconstructed direction

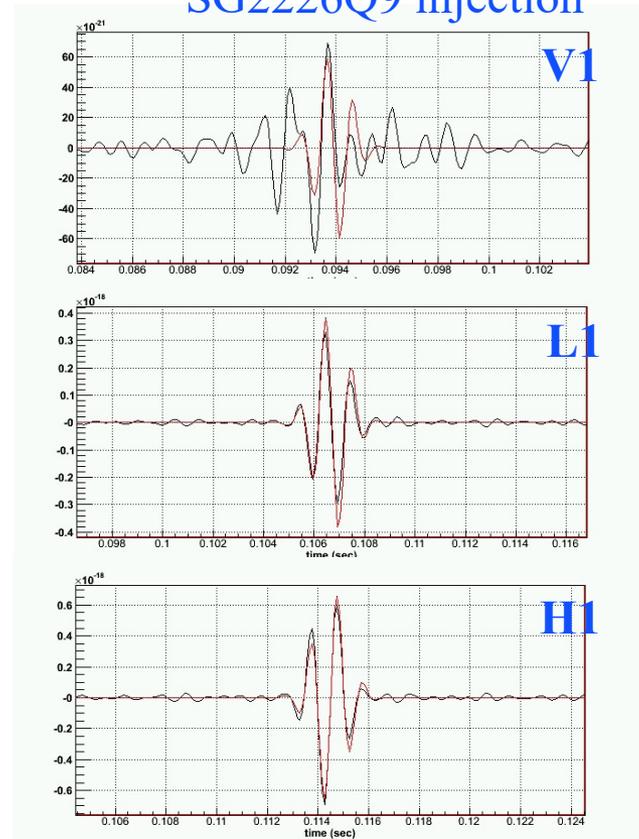
- The angular difference between injected and reconstructed position gives an estimation of the reconstruction error
- **Error Angle**: sum of sky pixel with likelihood greater than injected position
 - » Likelihood is used as a **ranking parameter**
 - » May be composed of disjoint areas in the sky



Waveform reconstruction

- The detector response vector in the DPF frame gives our solution
- From this solution we can recover the original detector response of each detector
 - » We reconstruct the GW signal for each detector
- Detector response can be confronted with source models for extraction of the source parameters

SG2226Q9 injection



Coherent Event Display

- CED is a detailed study of a particular event reporting more information than usual analysis

