Gravitational Waves and Cosmology

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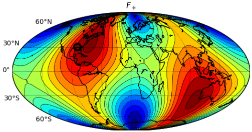
Almost omnidirectional detectors

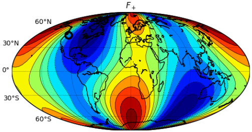
Detectors measure h_{det}: linear combination $F_+ h_+ + F_\times h_\times$

L H

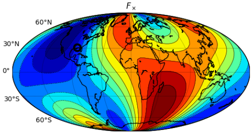
-1 0 1

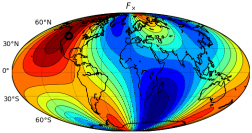
F_+





$F_\times$





$h_{+,\times}$ depend on source

pattern functions $F_{+,\times}$ depend on orientation source/detector

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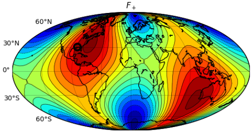
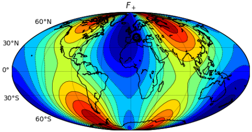
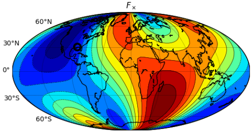
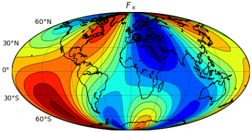




$F_\times$





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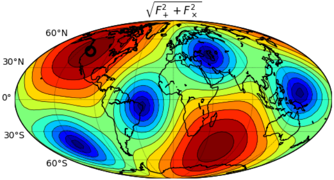
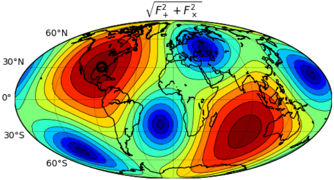
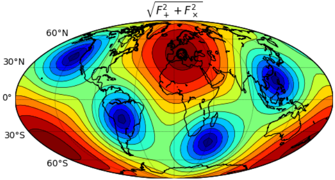
Pattern functions: $\sqrt{F_+^2 + F_\times^2}$







Wave generation: localized sources

Einstein formula relates h_{ij} to the source quadrupole moment Q_{ij}

$$
\begin{aligned}
Q_{ij} &= \int d^3x\rho\left(x_ix_j - \frac{1}{3}\delta_{ij}x^2\right), \qquad v^2 \simeq G_N M/r\,, \quad \eta \equiv m_1 m_2/M^2 \\
h_{ij} &\sim g(\theta_{LN})\frac{2G_N}{D}\frac{d^2Q_{ij}}{dt^2} \simeq \frac{2G_N\eta M v^2}{D}\cos(2\phi(t))
\end{aligned}
$$

$$
\begin{aligned}
f &= 2\mathrm{kHz}\left(\frac{r}{30\mathrm{Km}}\right)^{-3/2}\left(\frac{M}{3M_\odot}\right)^{1/2} < f_{Max} \simeq 12kHz\left(\frac{M}{3M_\odot}\right)^{-1} \\
v &= 0.3\left(\frac{f}{1\mathrm{kHz}}\right)^{1/3}\left(\frac{M}{M_\odot}\right)^{1/3} < \frac{1}{\sqrt{6}}
\end{aligned}
$$

Geometric factor $g(\theta_{LN})$ takes account of transversality projection (angular momentum L of the binary, observation direction N)

$$
\begin{aligned}
h_+ &\sim \frac{1+\cos^2(\theta_{LN})}{2}\,\eta\frac{Mv^2}{D}\cos\phi(t_s/M,\eta,S_i^2/m_i^4,\ldots) \\
h_\times &\sim \cos(\theta_{LN})\,\eta\frac{Mv^2}{D}\sin\phi(t_s/M,\eta,\ldots)
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Amplitudes of 2 polarizations modulated by θ_{LN} ($h \nearrow$ for $\theta_{LN} \searrow_0$), never both vanishing unlike dipolar motion for the electromagnetic case

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h_+ &\sim \frac{1+\cos^2(\theta_{LN})}{2}\eta\frac{\mathcal{M}v^2}{d_L}\cos\phi(t_O/\mathcal{M},\eta,S_i^2/m_i^4,\ldots) \\
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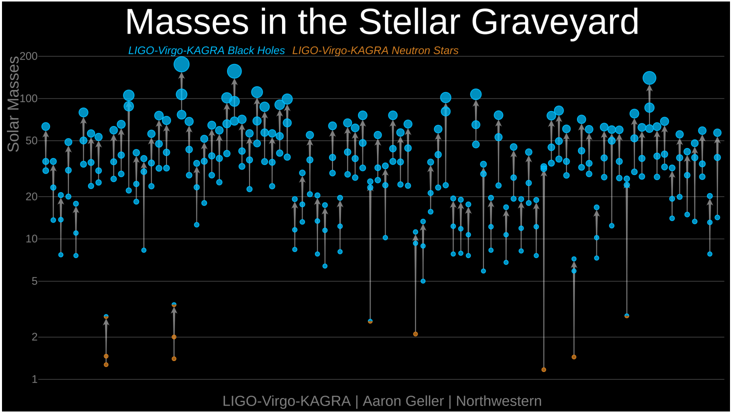
h sensitive to red-shifted masses $M \to M(1+z) \equiv \mathcal{M}$

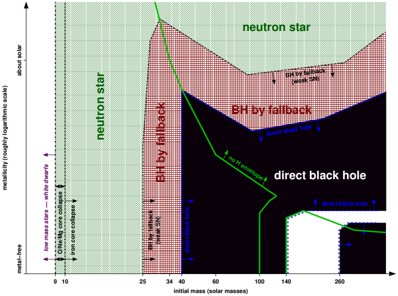
Stellar ($< 100 M_\odot$) compact object with known masses



Frequency 10-10^3 Hz determines size of sources
Remnant of GW190521 first Intermediate Mass Black Hole ($> 10^2 M_\odot$)
SuperMassive BHs $\gtrsim 10^5 M_\odot$ (up to $10^9 M_\odot$)

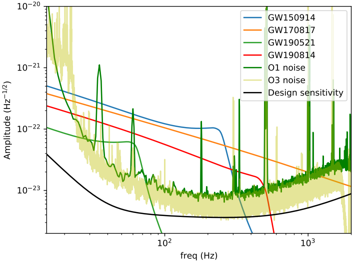
Solar Mass Black Holes

1st Mass gap: $2M_\odot < M_{BH} \lesssim 5M_\odot$
SN explosion prevents BH formation
2nd Mass gap: $50M_\odot < M_{BH} \lesssim 150M_\odot$
Pair Instability Super Nova
($\gamma \to e^+e^-$ drops pressure)
Are the (would be) gaps populated?



Heger+, Astrophys.J. 591 (2003) 288-300

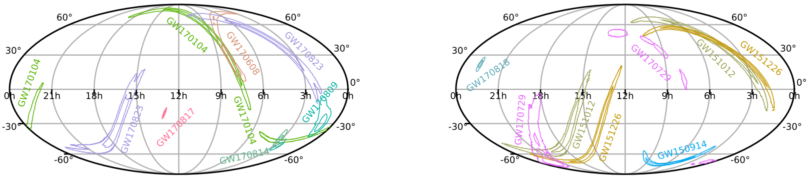
Sample wfs vs. detector's noise



Other groups searched for GWs in O1/2/3a public data:

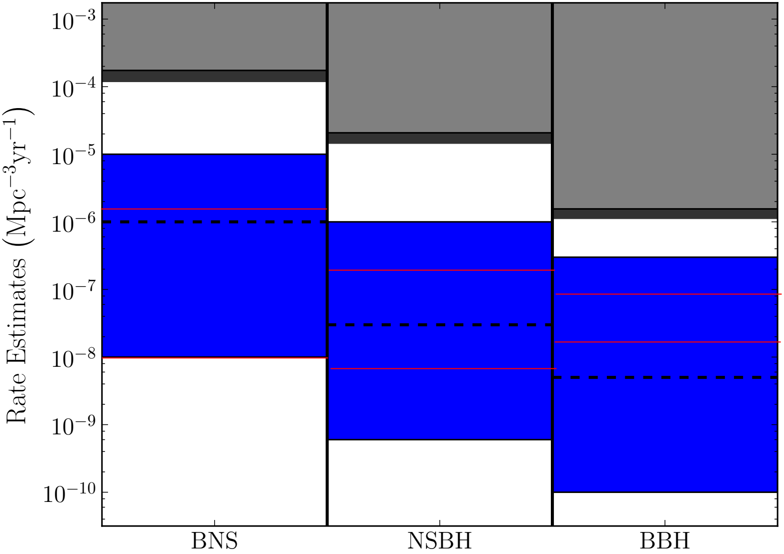
A. Nitz+ Ap.J. 891 123 ('19)
T.Venumadhav+ PRD 101 ('20)

Sky localization



Distances between 40 Mpc and ~ 5 Gpc ($\pm 20\%$)
(Milky Way's size $\sim 30\mathrm{kpc}$)

Image by Leo Singer, http://www.ligo.org

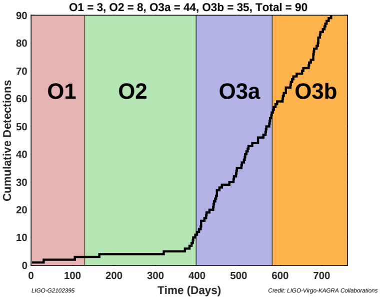


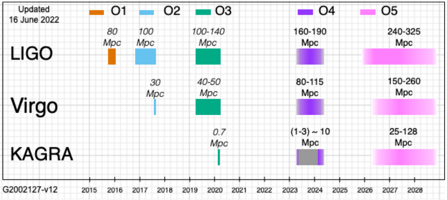
Astrophysical predictions, upper limit from observation from O1/O2/O3

Galaxy density $\sim 2 \times 10^{-2} Mpc^{-3}$

LIGO/Virgo COC ('10) 27 173001, PRX ('16), ApJ (2016), PRL 119 ('17)

LIGO/Virgo/KAGRA's prospects





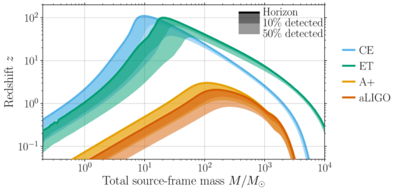
Future with ET and LISA looks very loud

Future 3rd generetaion detectors (Einstein Telescope, Cosmic Explore)/space telescope LISA will detect CBC signals with SNR $10 - 10^2$, with few golden events with SNR $\sim 10^3$.

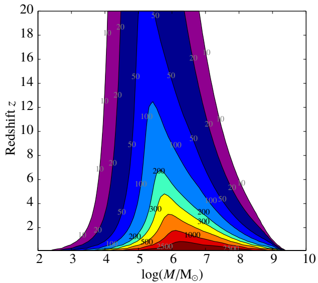
Templates few % accurate OK for characterising a source with SNR $O(10)$ (typical for LIGO/Virgo)

for SNR $\sim 10^3$ residual after extracting that source will have SNR $\sim O(10)$

1. baising parameter estimation
2. contaminating the extraction of additional sources.

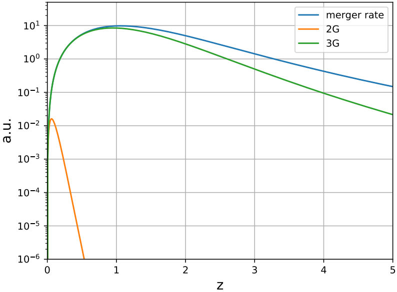


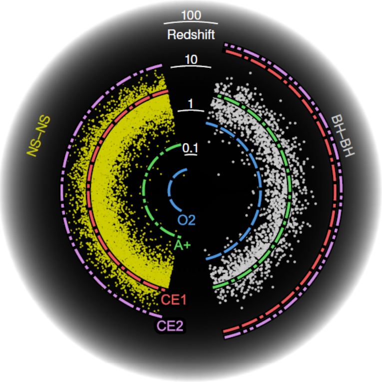
from arXiv:1902.09485



from arXiv:1201.3621

How many more?





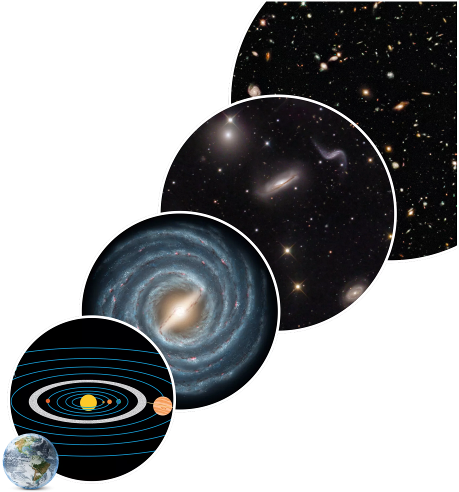
Leandro, Marra, RS PRD ’21

arXiv:1903.04615

New lines of research

- Precision gravity
- General Relativity tests in strong gravity
- "Hairiness" of black holes
- Universe expansion history (standard sirens)
- Phase transitions in the Early Universe
- Black hole mass function
- How binary systems form and how frequent are they? (star formation rate)
- Fate of a massive star
- Probe the the interior of a neutron star
- Understand the life of a pulsar and its evolution
- Dark matter and Gravitational Waves (modification of compact object and/or their environment)
- Data analysis challenges for signals with $SNR \sim 100$
- Central Core of Galaxies, Massive Black Holes and their role in Galaxy Formation
- Multi-messenger astronomy

Cosmic ladder



$$6 \text{ thousand km} \overset{10^6}{\rightarrow} 10 \text{ billions km} \overset{10^7}{\rightarrow} 15 \text{ kpc} \overset{10^5}{\rightarrow} 1 \text{ Mpc} \overset{10^3}{\rightarrow} 3\text{Gpc}$$

$$1\text{kpc} \simeq 3 \times 10^{16} \text{ km}, \; 3\text{kpc} \sim 10^4 \text{ light year}$$

GW and cosmology

In GR the luminosity distance is related to red-shift via the matter content of the Universe $ds^2 = dt^2 - a^2(t)d\vec{x}^2$, $a \equiv 1/(1+z)$

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8}{3}\pi G\rho\,, \qquad d_c = \Delta x = \int \frac{dt}{a(t)} = \int \frac{da}{a^2}\frac{a}{\dot{a}} = \int \frac{dz}{H(z)}$$

EM observed:

$$d_L \equiv \left(\frac{\dot{E}}{Flux}\right)^{1/2}$$

$$d_L = \frac{1+z}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda(1+z)^{3(1+w_\Lambda)}}} \simeq \frac{z}{H_0}$$

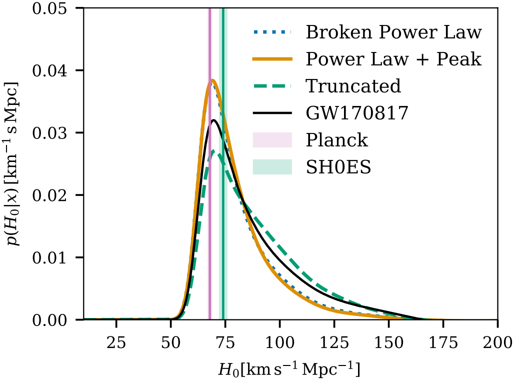
where $\Omega_\Lambda = \rho_\Lambda/\rho_0$ is the still misterious *dark energy* and $w_\Lambda \equiv \frac{p_\Lambda}{\rho_\Lambda} \simeq -1$
$\Omega_m \equiv \frac{\rho_m}{\rho_0}$

GWs are *standard sirens*, with calibrated d_L

- Need for an EM counterpart to know z and/or *complete* galaxy catalog
- Use of average properties of galaxy populations
- Possibility of model independent measure of equation of state of the dark energy

The importance to know distance and redshift

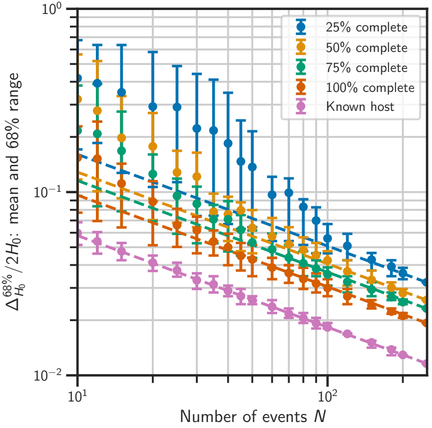
Luminosity distance vs. redshift: $D_L H_0 = z + O(z^2)$



H_0 determination from EM bright 1 standard candle and 46 dark ones, short-circuiting with galaxy survey catalog GLADE+ Dálya et al. arXiv:2110.06184

LIGO/Virgo/KAGRA arXiv:2111.03604

Bright/dark Sirens with 2G observatories



R. Gray+, arXiv:1908.06050

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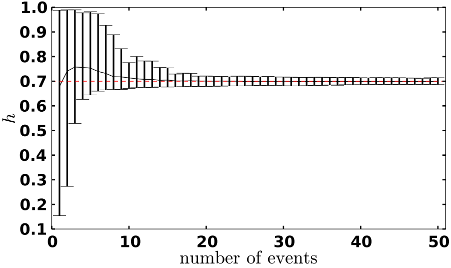
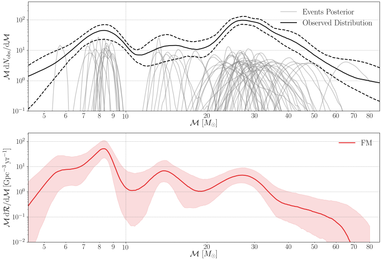


Del Pozzo 1108.1317

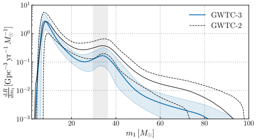
Population inference

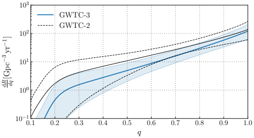
$$\dot{f} = \frac{\dot{E}}{\frac{dE}{df}} \simeq \frac{\eta^2 v^{10}}{\eta M v}\frac{df}{dv} \simeq \eta M^{5/3} f^{11/3}\left(1 + v^2 \#(\eta)\dots\right)$$

with $v = (G_N \pi M f)^{1/3}$ (Kepler law)
M_c inference:



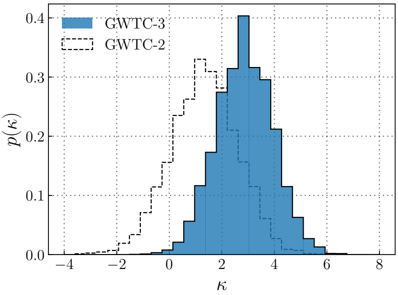
m_1 and $q \equiv m_1/m_2$ inference:

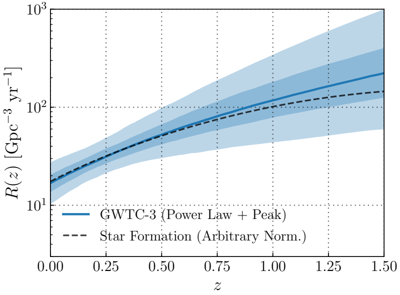




Rate vs. z

$R \propto (1+z)^{\kappa}$:





Cosmological Bayesian inference

Aiming at cosmological parameter's posteriors f (for out model: H_0,Ω_m)

$$f(H_0) = p(H_0)p(\Omega_m)p(z|H_0,\Omega_m)\frac{\mathcal{L}(d_L|H_0,\Omega_m,z)}{\mathcal{E}}$$

In absence of a redshift measure, the z prior is crucial

$$p(z|H_0,\Omega_m) = \underbrace{A(H_0,\Omega_m)}_{normalization}\underbrace{R_m^{(\{\theta_i\})}(z)}_{merger\ \ rate}\underbrace{f_{det}(d_C^{(t)}(z))}_{detector}$$

Merger rate $R_m \sim$ star formation rate $\mathcal{R}_f$ + *Poissonian* delay

$$R_m^{(\tau)}(z) = \int_0^z dz_f \frac{dt}{dz_f}\mathcal{R}_f(z_f)\exp\left(-\frac{t(z_f)-t(z)}{\tau}\right)$$

S.Vitale, W. M. Farr, K. Ng, C. L. Rodriguez, arXiv:1808.00901, APJL '19

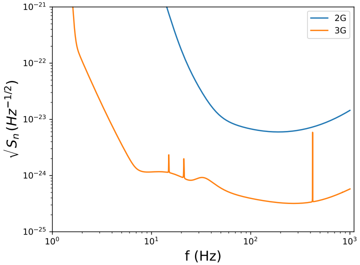
$$\mathcal{R}_f \propto \frac{(1+z)^{2.7}}{1+\frac{1+z}{2.9}^{5.6}}$$

P. Madau, M. Dickinson, arXiv:1403.0007, Ann. Rev. A.A. '14

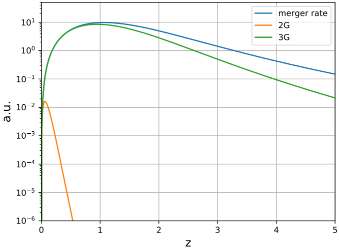
see also X. Ding et al. arXiv:1801.05073 JCAP '19

Detector acceptance

Detector sensitive to $SNR = 2\left(\int_0^\infty \frac{|\tilde{h}(f)|^2}{S_n}\right)^{1/2} \geq 8$



Noise spectral density



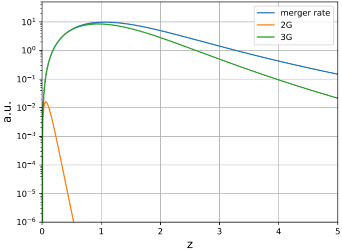
Distribution of detected events ($\tau = 5\,Gyr$)

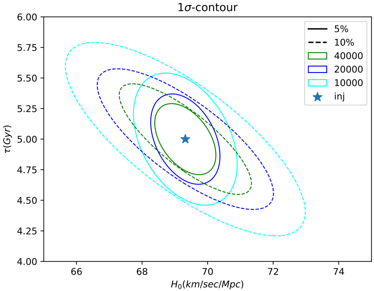
Typical sources solar mass BHs up to $\sim 10^{2(3)} M_\odot$ for 2(3)G

Exp. cutoff at $d_L \sim$ 320Mpc (2G, $z \sim 0.1$), $d_L \sim$ 7.9 Gpc (3G, $z \sim 1.2$)

Black sirens with 3G observatories

Information also stored in black sirens if *statistical distribution* of merger known (with hyper-parameter τ)





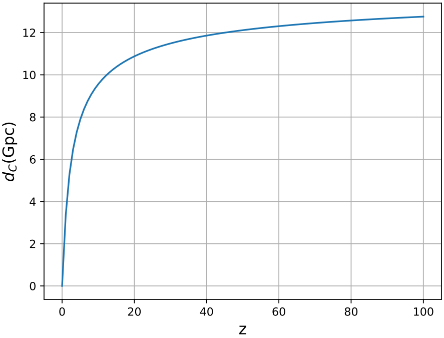
Worst prior knowledge of the redshift distribution (modeling merger rate with more hyper-parameters) degrades predictive power of cosmo pars

Opportunity: fit cosmology and population property

H. Leandro, V. Marra, RS PRD '21

z-dependence of SNR

GW amplitude $|\tilde{h}(f)| \sim \frac{(M(1+z))^{5/6} f^{-7/6}}{d_c(1+z)}$



d_c levels off at $\sim 12 Gpc$ ($z \sim 40$), $\implies$ SNR almost independent on z at large distances until signal maximum frequency drops out of the bandwidth

$$f_{max} \sim 20 Hz \left(\frac{M(1+z)}{10^3 M_\odot} \right)^{-1}$$

Conclusion

- Gravitational Wave Astronomy is a young and fast growing science, its impact will go beyond astronomy
- Strong interplay between cosmology/astronomy

The LIGO and Virgo observatories

• Observation run **O1** Sept '15 - Jan '16
$\sim$ 130 days, with 49.6 days of actual data, PRX (2016) 4, 041014, 2 detectors, 3BBH

• **O2** Dec. '16 – Jul'17 2 det's + Aug '17 3 det's
3(+4) BBH + 1BNS in double (triple) coinc.

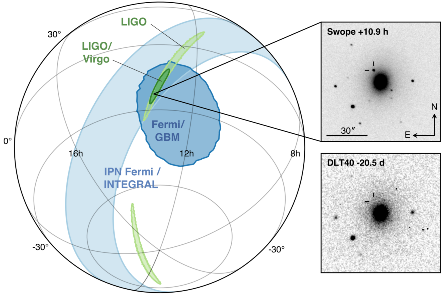
• **O3a**: 3 detectors, Apr - Sep 2019, 39 dets

• **O3b**: Nov 1st – Mar 27th 2020 $\rightarrow$ 90 detections

In April 2020 KAGRA joined, in 202? INDIGO

GW170817

- GW trigger on Aug 17th, 2017, ended at 12h 41' 04.4" UTC, first in in LIGO Hanford, then confirmed as a triple coincidence $\rightarrow$ localized in an area of $\sim$ 28 deg^2
- GRB trigger from Fermi-GBM 1.7" after
- first optical image 10.87 hr afterwards by One-Meter Two Hemisphere team with Swope telescope at Las Campanas Observatory in Chile
- X obtained by the X-Ray Telescope on Swift after 14.9 h (NuSTAR 16.8 h)
- radio ($\sim 3, 6$ GHz) by VLA 16 days after GW event



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