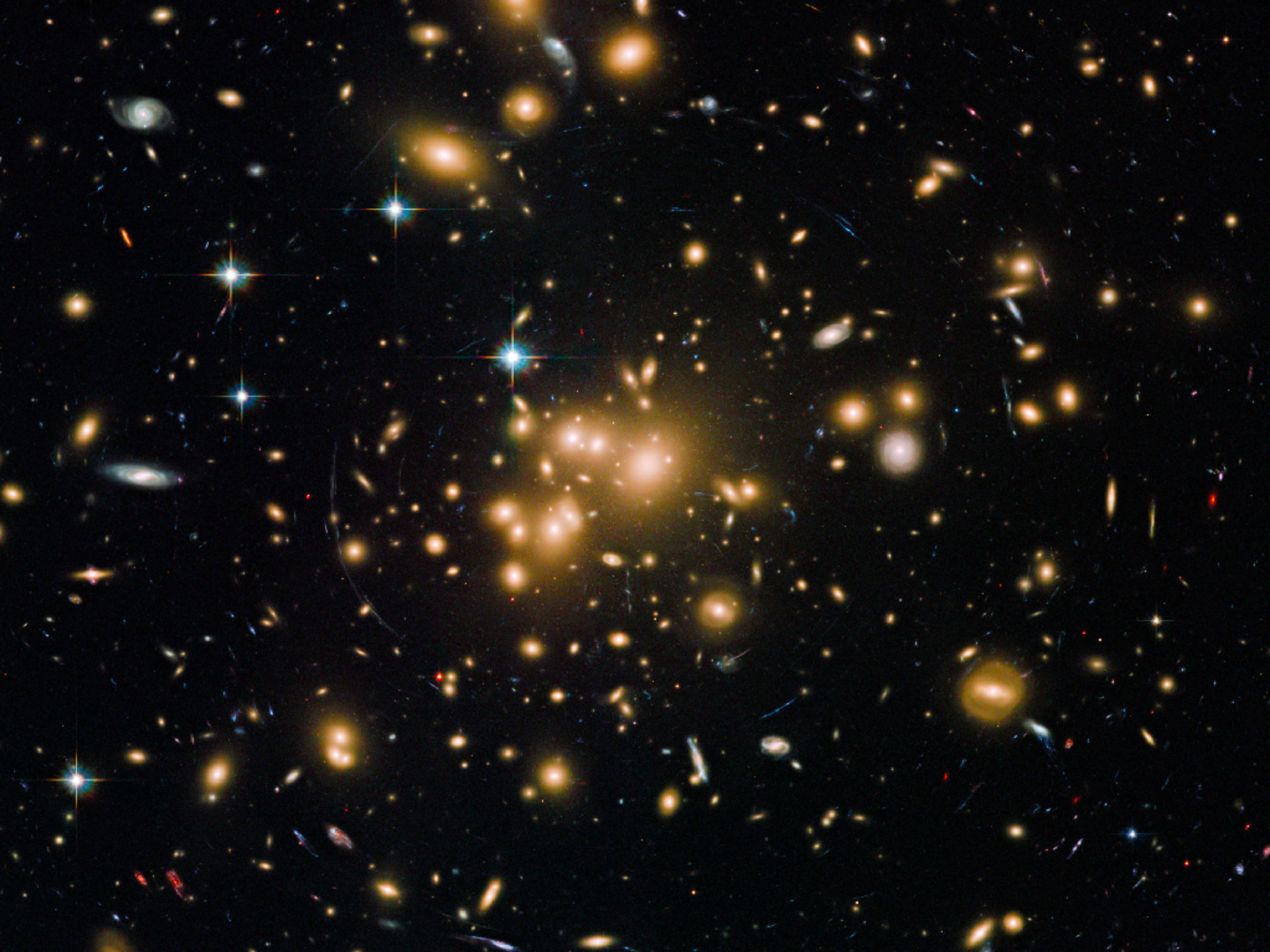


# GRAVITATIONALLY LENSED TRANSIENTS AND TRANSIENTS FROM GRAVITATIONAL LENSING

MARTÍN MAKLER
ICAS/IFICI/CONICET&UNSAM AND CBPF

UNSAM

ICAS

CONICET

ICIFI

CBPF

PPGCosmo

São Paulo Advanced School on
**Multi-Messenger Astrophysics**

INSTITUTO PRINCIPIA

# Lensing in the Context of Mutlimessenger Astrophysics

Martín Makler
ICAS/IFICI/CONICET&UNSAM and CBPF

UNSAM ICAS CONICET ICIFI CBPF PPGCosmo

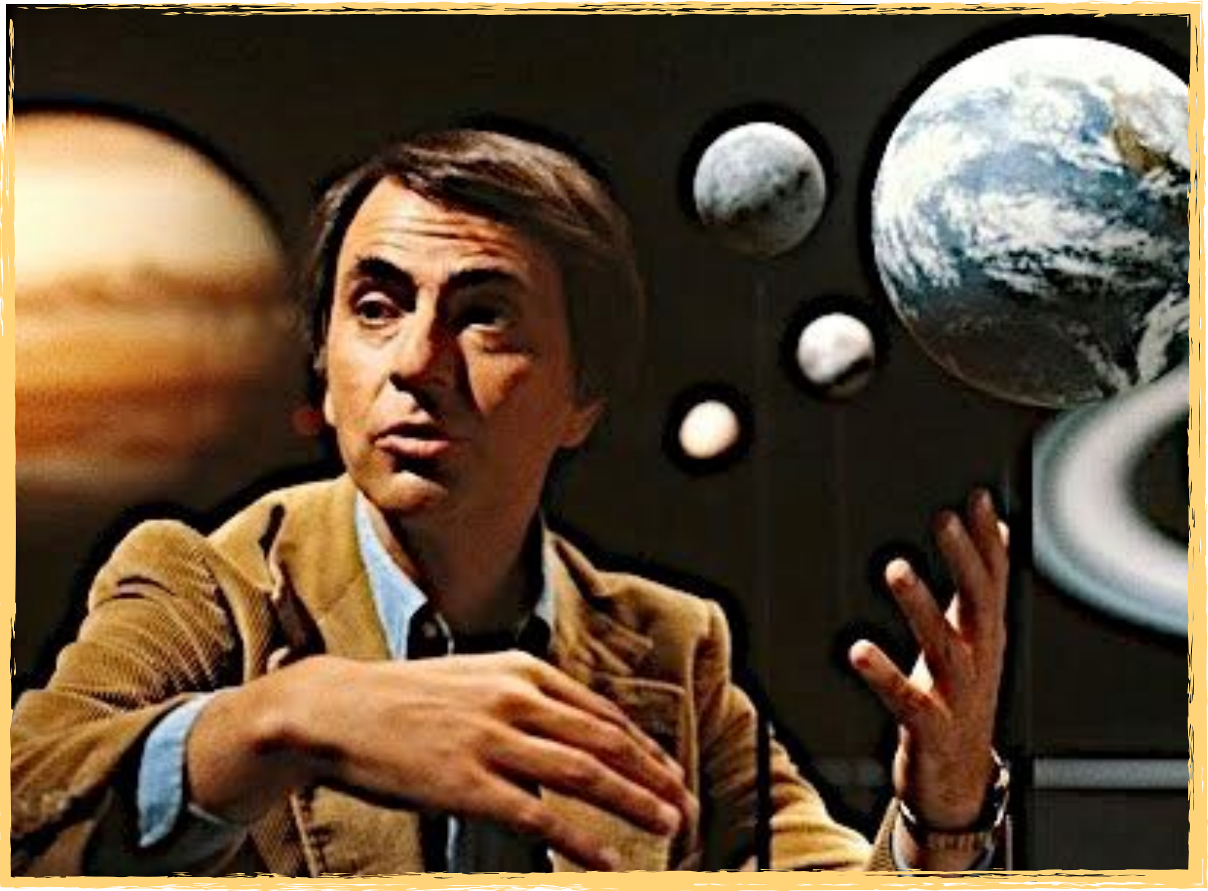
São Paulo Advanced School on
**Multi-Messenger Astrophysics**

INSTITUTO PRINCIPIA

# OUTLINE

- Basics of gravitational lensing (photons)
- Geometrical optics
  - magnification, point sources: microlensing
  - multiple images: strong lensing
- Time delay
  - quasars and supernovae
- Diffractive gravitational lensing
  - primordial black holes
- Lensing of gravitational waves
  - With and without EM counterparts
- Concluding remarks

# Questions welcome!



“Every question is a cry to understand the world. There is no such thing as a dumb question”

*Carl Sagan*

*(The Demon-Haunted World: Science as a Candle in the Dark)*

# Warning: don’t panic!

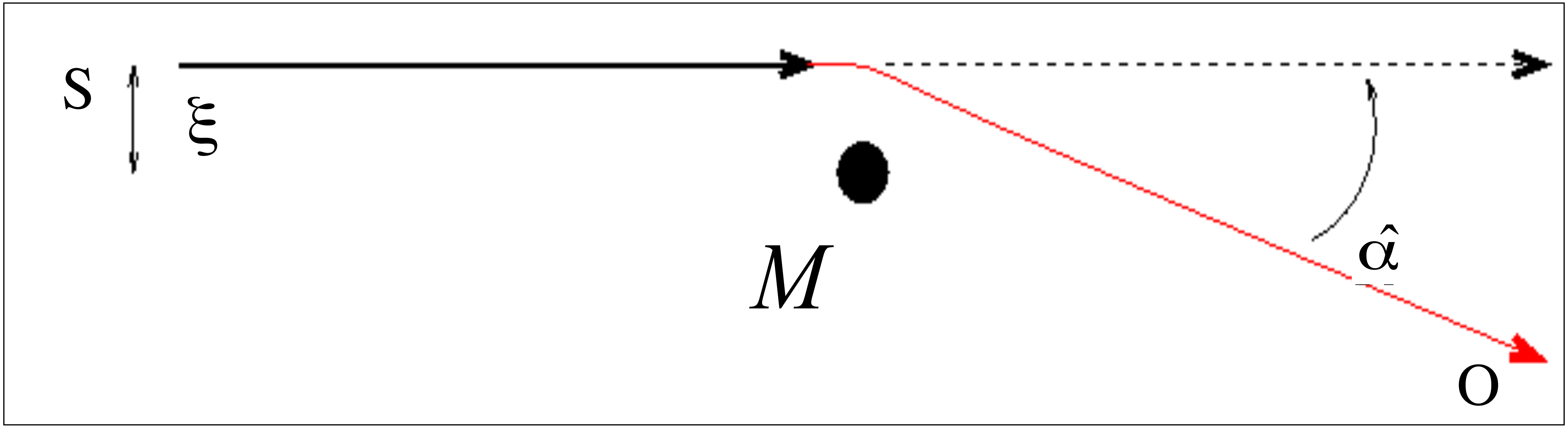
- I will try to provide an overview of a broad subject: tons of concepts
- Since this is a school, I’ll try to give a peek at some of the math/physics
    - Some equations and notations will be flashed out
- Don’t worry if you don’t understand them, get the concepts
- Will not address any technical problems (oversimplification)
- At least you know what to look for over there!
- Check the references for details
- Talk to me at any time during the school

# BENDING OF LIGHT BY GRAVITY

Null geodesic,
Fermat principle

$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\phi}{c^2}\right) d\sigma^2$$

$$\frac{d\sigma}{dt} := c' = \sqrt{\frac{1 + 2\phi/c^2}{1 - 2\phi/c^2}} \simeq c\left(1 + \frac{2\phi}{c^2}\right)$$



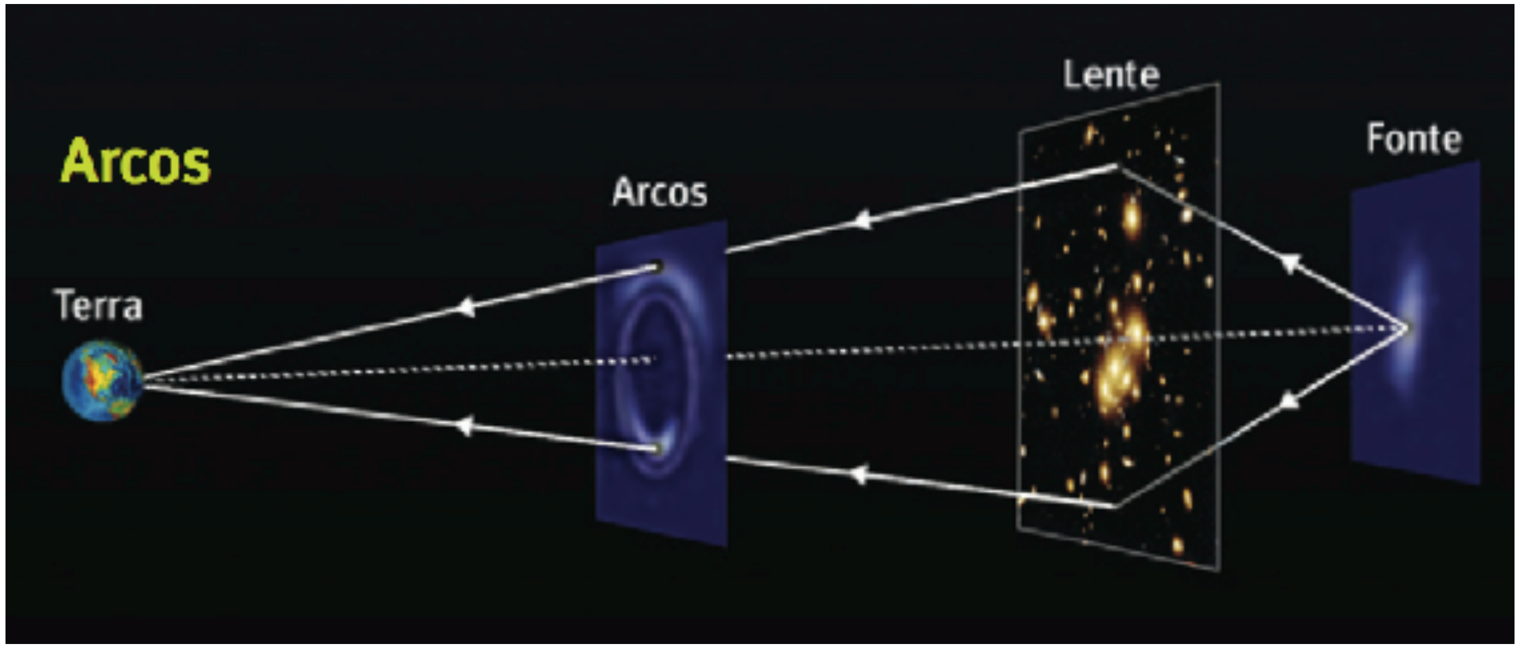
Deflection angle: $\hat{\alpha} = \frac{4GM}{c^2}\frac{1}{\xi}$

# STRONG LENSING

- Gravitational lensing (geometrical optics): null geodesics
  - surface brightness conservation
  - achromatic

  } → Gravitational telescopes

  - Unique probe of the mass distribution in galaxies and clusters ➔ DM, *b*
  - Provide complementary cosmological probes and tests of gravity
- Strong Lensing: multiple images, strong distortions, large magnifications, time delays



strong lensing, **weak gravity**

Gravitational arcs

# A PLETHORA OF LENSING PHENOMENA

Strength

- Strong lensing
  - Strong magnifications
  - Multiple images
  - Distortions
    - Rings
    - Arcs
- Weak Lensing
  - Small twist
  - Small magnification
  - Detected statistically

Angular scale

- Micro-lensing
  - MACHOS
  - Planetary search
- Micro and mili-lensing
  - Quasars
- “Macro-lensing”
  - Galaxies
  - Clusters
  - Large-scale structure

+ astrometric microlensing, black-hole shadows, retrolensing, femtolensing, lensing of gravitational waves….

# A PLETHORA OF LENSING PHENOMENA

Strength

Angular scal

- Strong lensin
  - 
  - I
  - D
    - –
    - – A

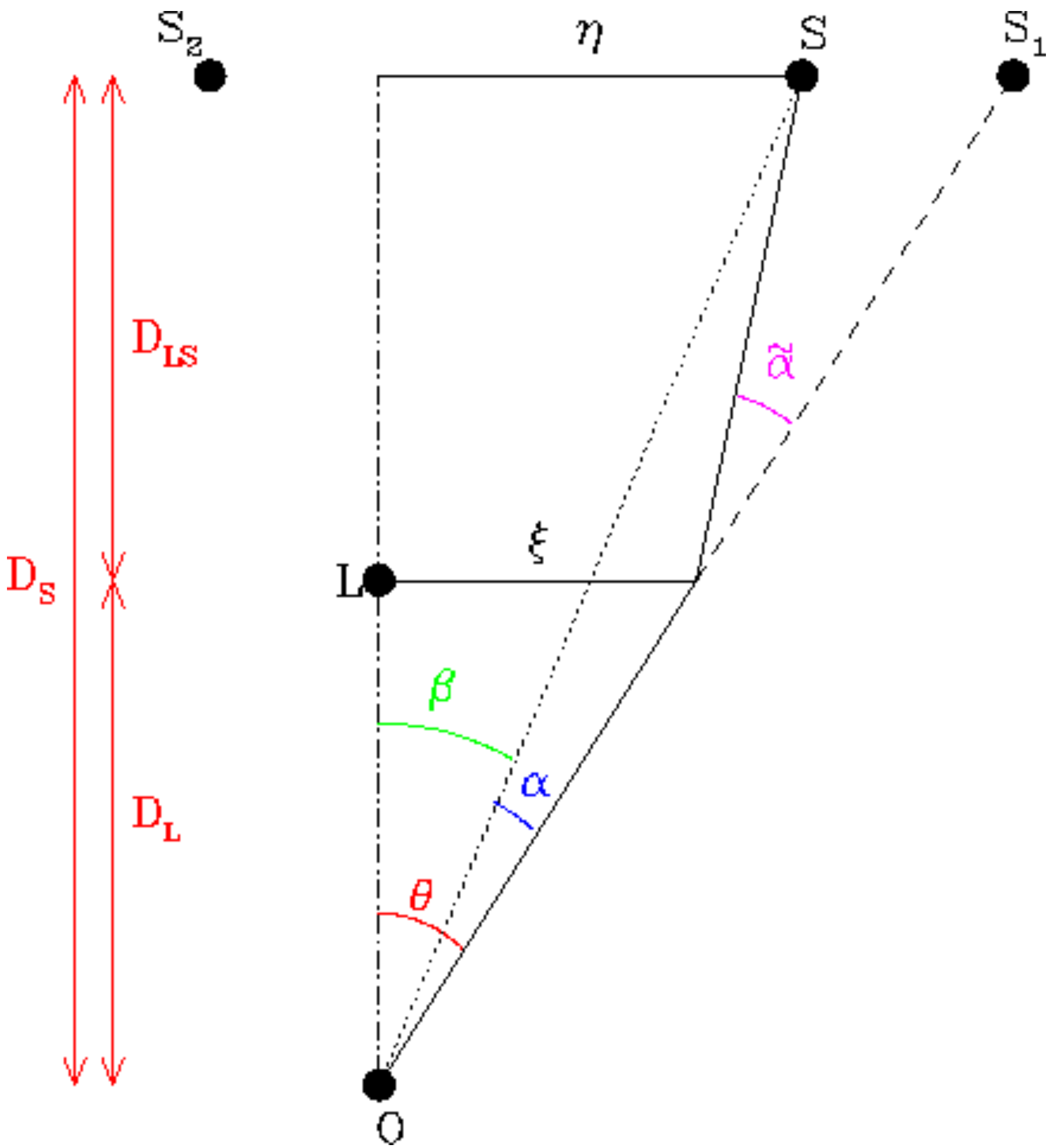
ing

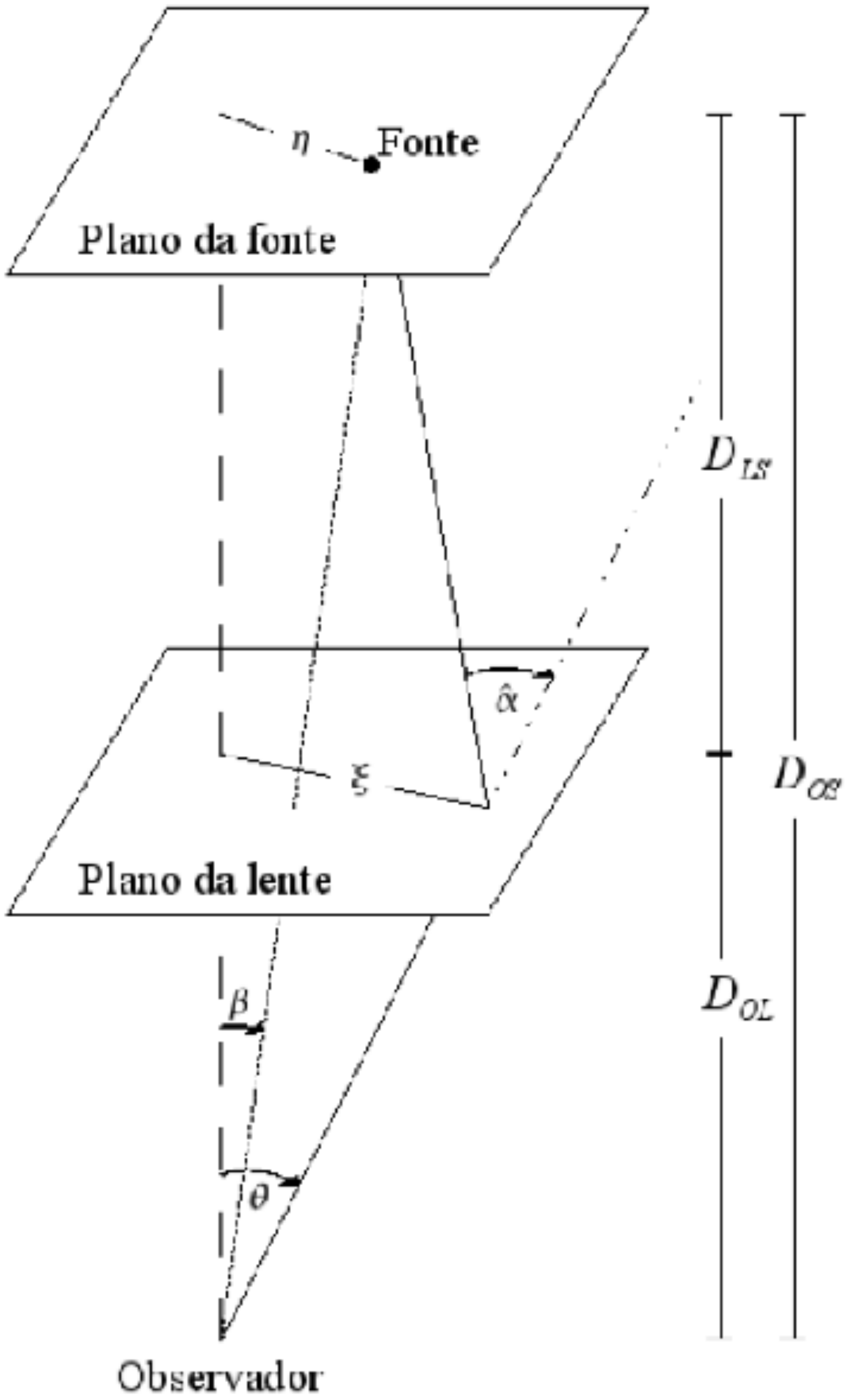
- Weak
  - Sm
  - Sma
  - Det

+ astro microlensing, black-hole shadows, retrolensing, femtolensing, lensing of gravitational waves….

**Gravitational Lensing is a physical phenomenon, with many techniques, which enables a lot of interesting science**

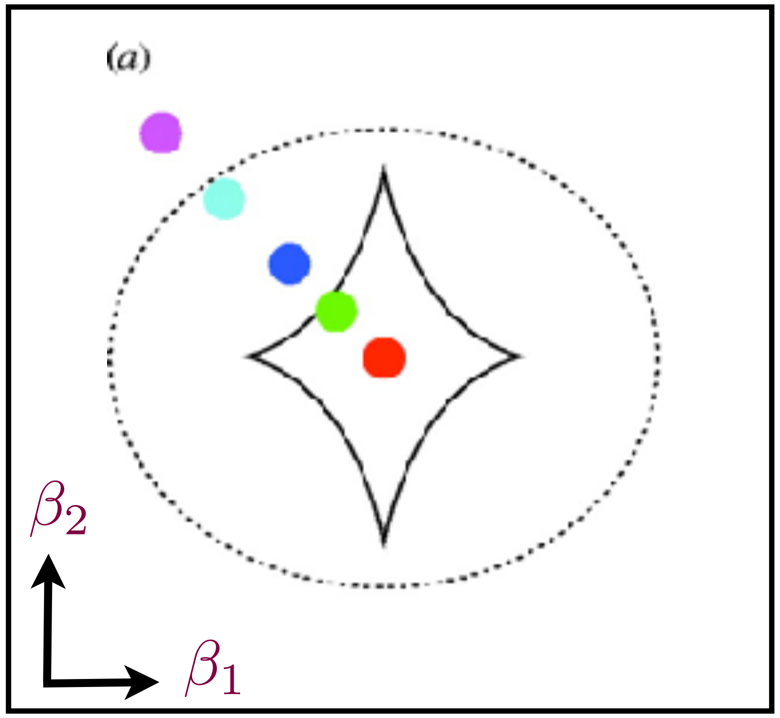
# Geometry of lensing by a single plane



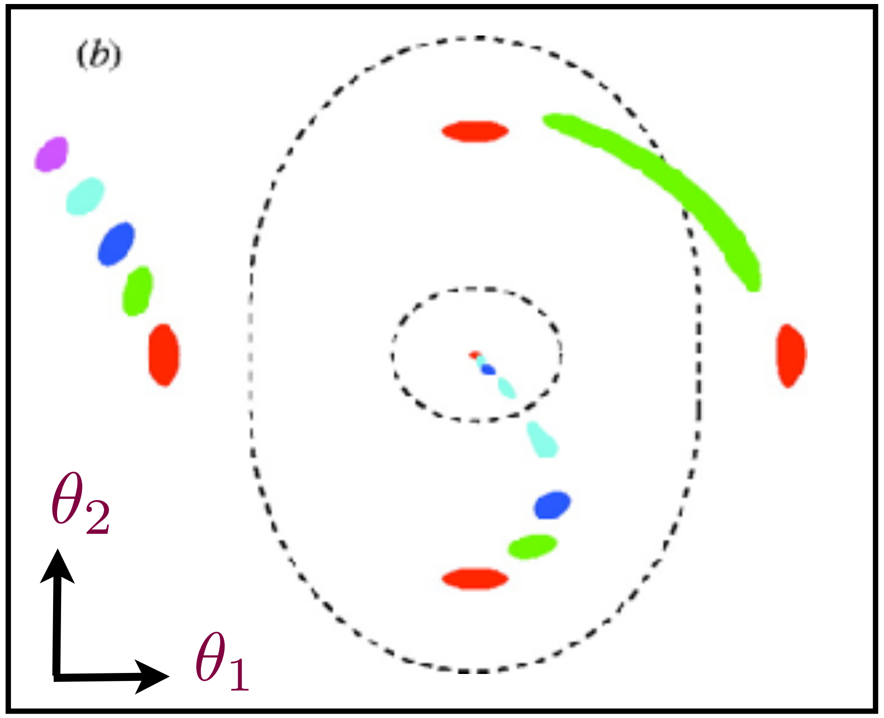


# Geometry of lensing by a single plane

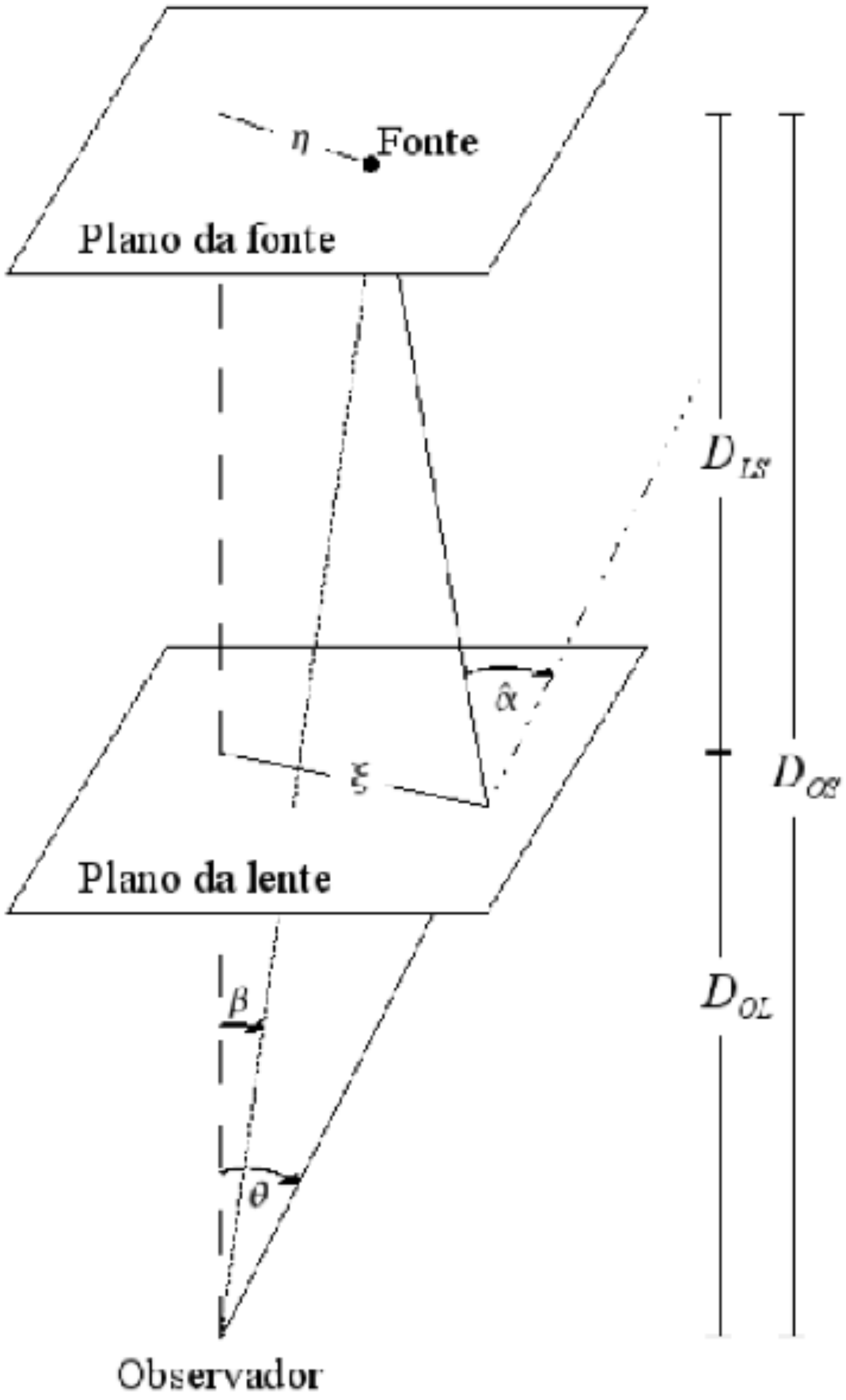
(assumptions: weak field, single plane, geometrical optics)



Source plane

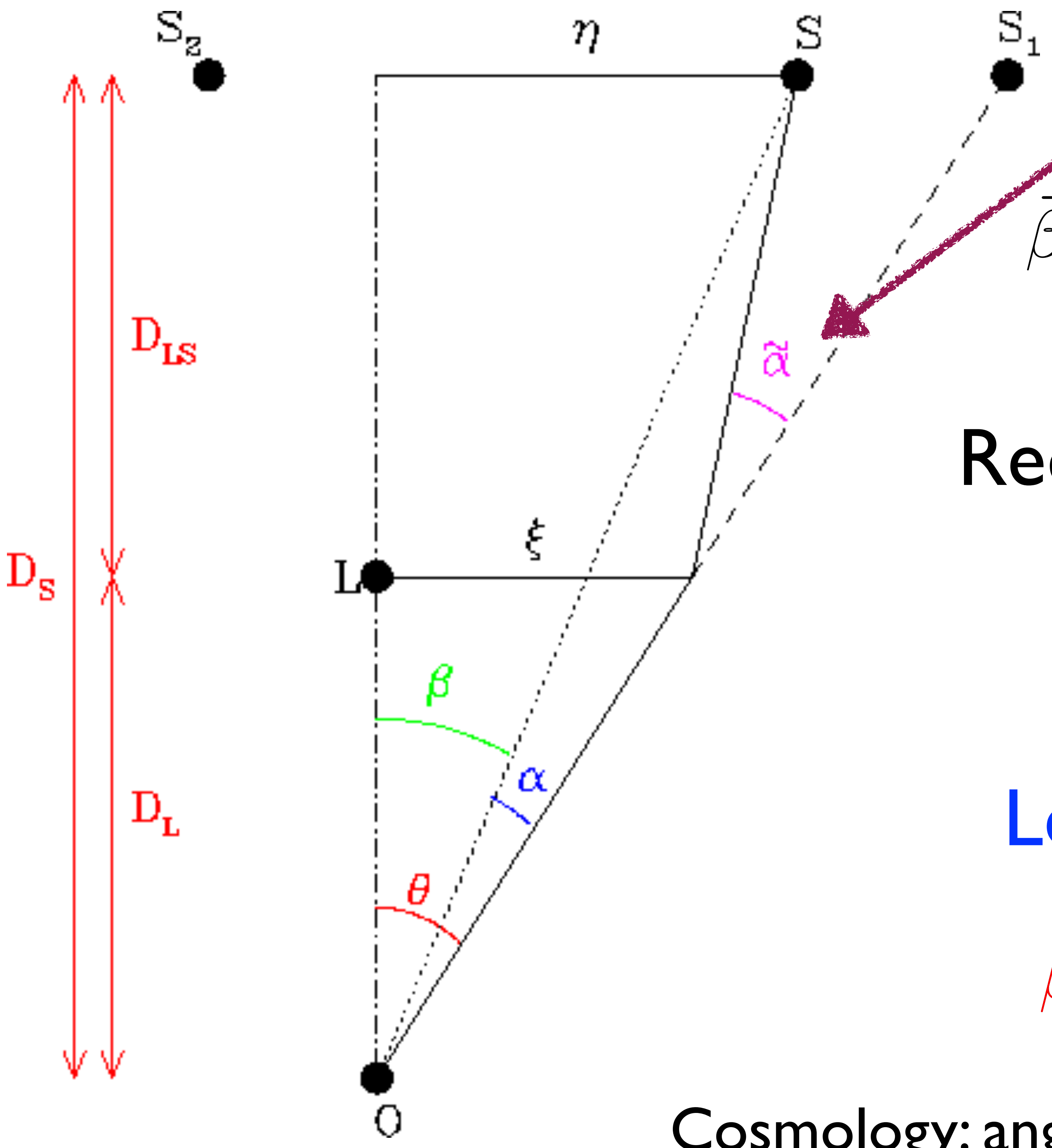


Lens plane



| | | |
|---|---|---|
| Physical coordinates (distance) | $\eta$ | $\xi$ |
| Angular coordinates | $\beta$ | $\theta$ |
| dimensionless coordinates | $y$ | $x$ |

# The Lens Equation



The physics is in here

$$\vec{\beta} D_{OS} = \vec{\theta} D_{OS} - \hat{\vec{\alpha}} D_{LS} \left( \vec{\theta} \right)$$

Reduced deflection angle

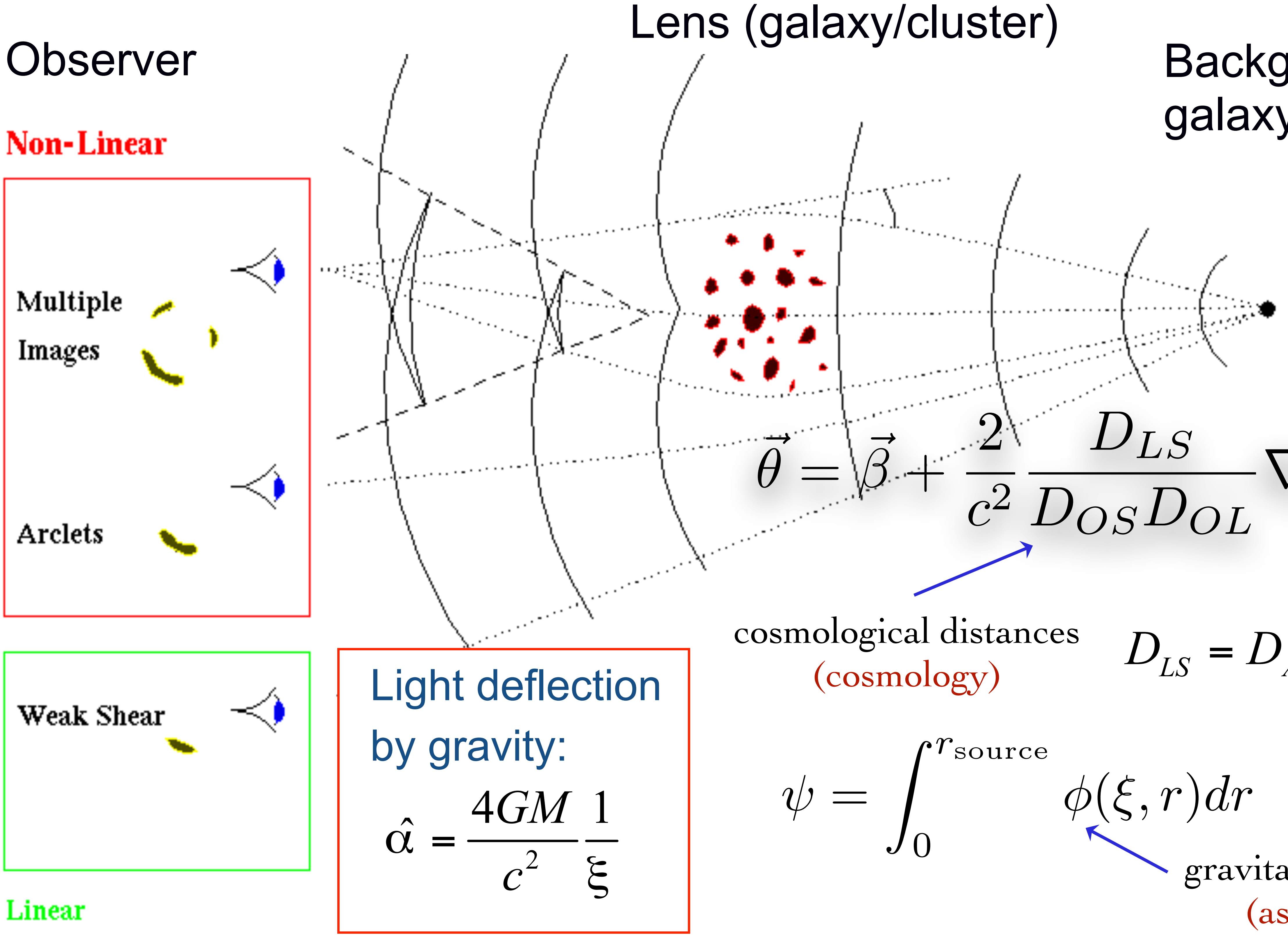
$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}}$$

Lens equation

$$\vec{\beta} = \vec{\theta} - \vec{\alpha} \left( \vec{\theta} \right)$$

Cosmology: angular diameter distances

# Weak and Strong Lensing Effects



$$\vec{\theta} = \vec{\beta} + \frac{2}{c^2}\frac{D_{LS}}{D_{OS}D_{OL}}\nabla_\theta \psi\left(\vec{\theta}\right)$$

cosmological distances
(cosmology)

$$D_{LS} = D_A\left(z_L, z_S\right)\ldots$$

Light deflection by gravity:

$$\hat{\alpha} = \frac{4GM}{c^2}\frac{1}{\xi}$$

$$\psi = \int_0^{r_{\mathrm{source}}} \phi(\xi, r)dr$$

gravitational potential
(astrophysics)

# Reminder:
# Angular Diameter Distance

In the wCDM model $p = w\rho$

$$H^2(a) = H_0^2 \left[ \Omega_r a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_{DE} a^{-3(1+w)} \right]$$

In the flat case

$$D_A\left(z_1, z_2\right) = \frac{\left(1+z_2\right)^{-1}}{H_0} \int_{z_1}^{z_2} \frac{dz'}{\sqrt{\Omega_M (1+z')^3 + \left(1-\Omega_M\right)\left(1+z'\right)^{3(1+w)}}}$$

$$D_{LS} = D_A(z_L, z_S)$$

# Lensing by a point lens

Magnification and microlensing

# Point Mass Lens

Point mass $$\hat{\vec{\alpha}} = \frac{4GM}{c^2\xi}$$

Reduced deflection angle $$\vec{\alpha} = \frac{D_{LS}}{D_{OS}D_{OL}}\frac{4GM}{c^2\theta}$$

Lens equation $$\beta = \theta - \frac{\theta_E^2}{\theta}$$

Einstein angle $$\theta_E = \sqrt{\frac{D_{LS}}{D_{OS}D_{OL}}\frac{4GM}{c^2}}$$

# Images and magnification

Lens equation
$$\beta = \theta - \frac{\theta_E^2}{\theta}$$

Solutions
$$\theta_{1,2} = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta^2}\right)$$

Magnification
$$\mu = \frac{\theta}{\beta}\frac{d\theta}{d\beta}$$

# Images and magnification

Lens equation $$\beta = \theta - \frac{\theta_E^2}{\theta}$$

Solutions $$\theta_{1,2} = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta^2}\right)$$

Magnification $$\mu_{1,2} = \left(1 - \left[\frac{\theta_E}{\theta_{1,2}}\right]^4\right)^{-1} = \frac{1}{2} \pm \frac{u^2+2}{2u\sqrt{u^2+4}}$$

Distance in units of the Einstein angle $$u = \beta/\theta_E$$

Total magnification $$\mu = |\mu_1| + |\mu_2| = \frac{u^2+2}{u\sqrt{u^2+4}}$$

# Microlensing light curve

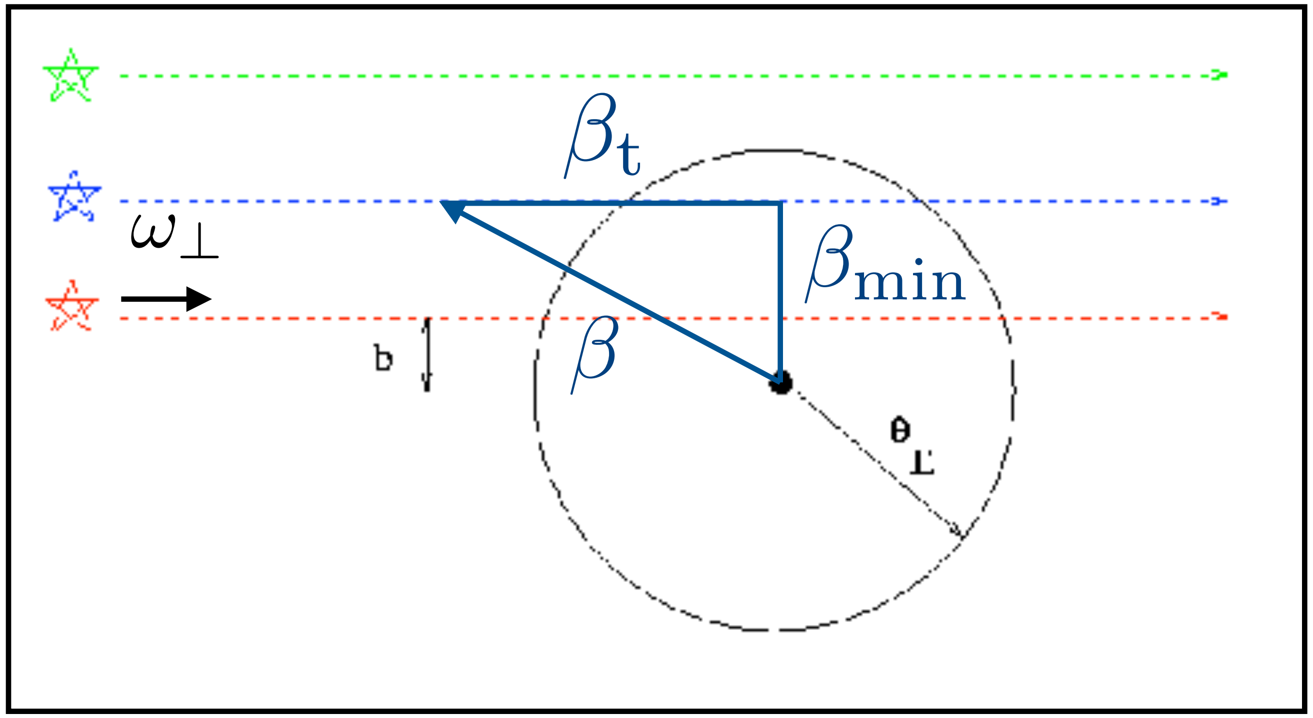
Point lens point source magnification:

$$\mu = \frac{u^2+2}{u\sqrt{u^2+4}} \qquad u = \beta/\theta_E$$

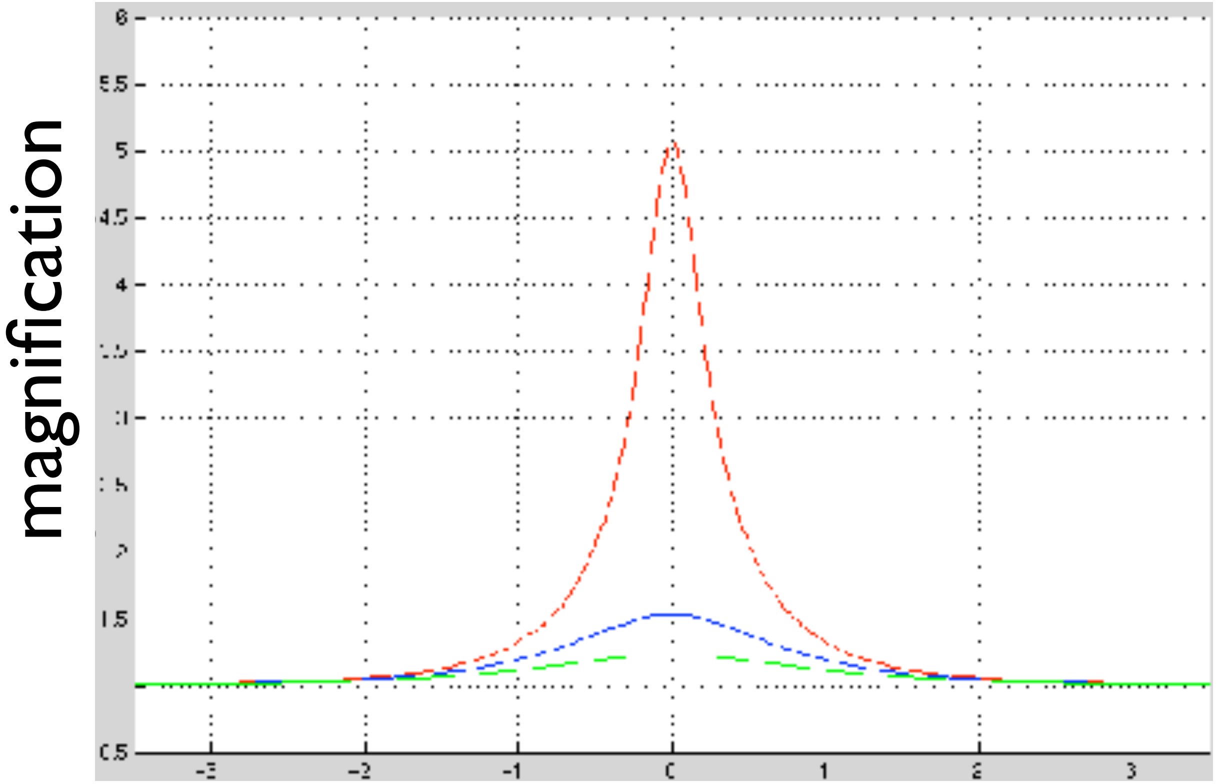
Einstein radius (angle):

$$\theta_E = \sqrt{\frac{D_{LS}}{D_{OS}D_{OL}}\frac{4GM}{c^2}}$$

$\omega_\perp$ : angular velocity between lens and source



Source plane



Time in Einstein time units

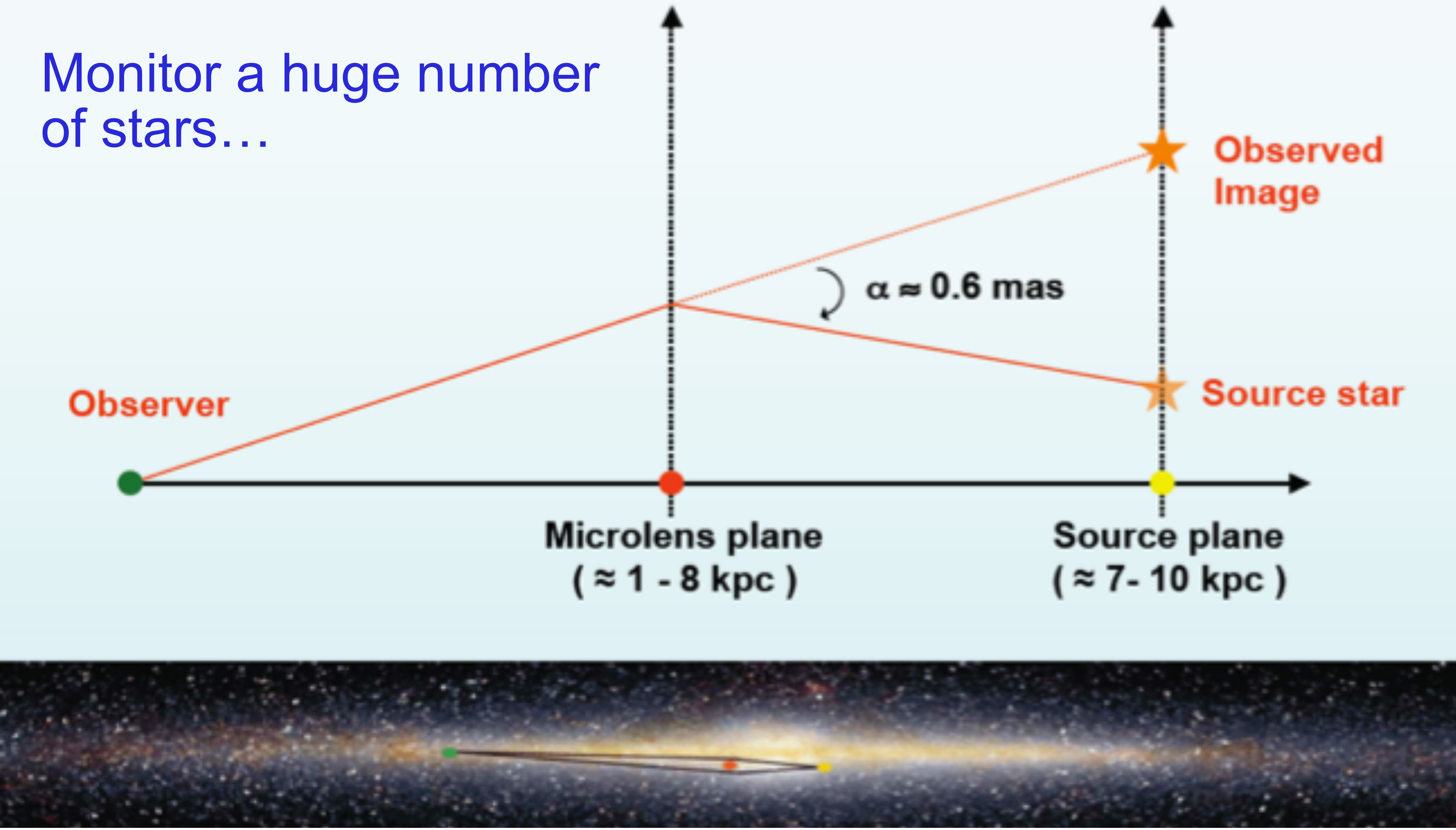
$$\beta_t = \omega_\perp t = \frac{v_\perp}{D_{OL}} t$$

$$u(t) = \sqrt{u_{\rm min}^2 + \left(\frac{v_\perp t}{\theta_E D_{OL}}\right)^2} = \sqrt{u_{\rm min}^2 + \left(\frac{t}{t_E}\right)^2}$$

$$\boxed{t_E = \frac{\theta_E D_{OL}}{v_\perp}}$$

# Galactic microlensing

Monitor a huge number of stars…



# What is microlensing? *Usually*, no spatial information

$$\mu = \frac{u^2+2}{u\sqrt{u^2+4}}$$

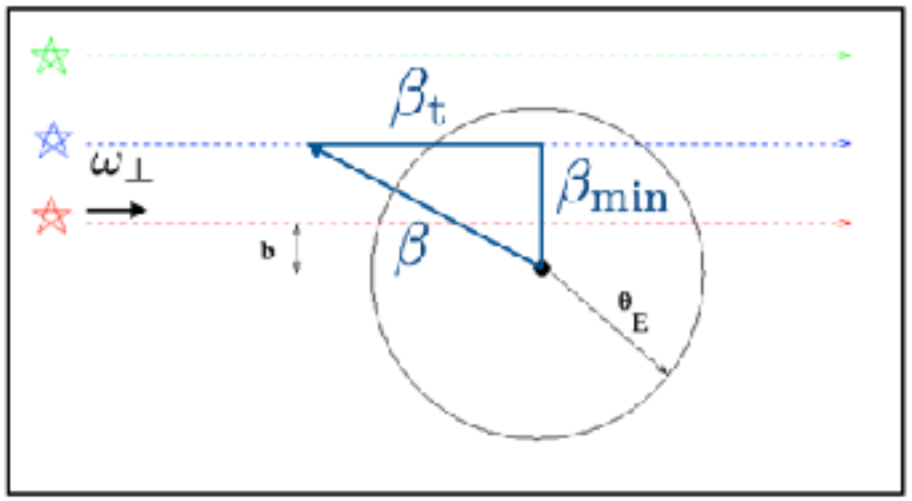
$$u = \beta/\theta_E$$

Einstein Angle

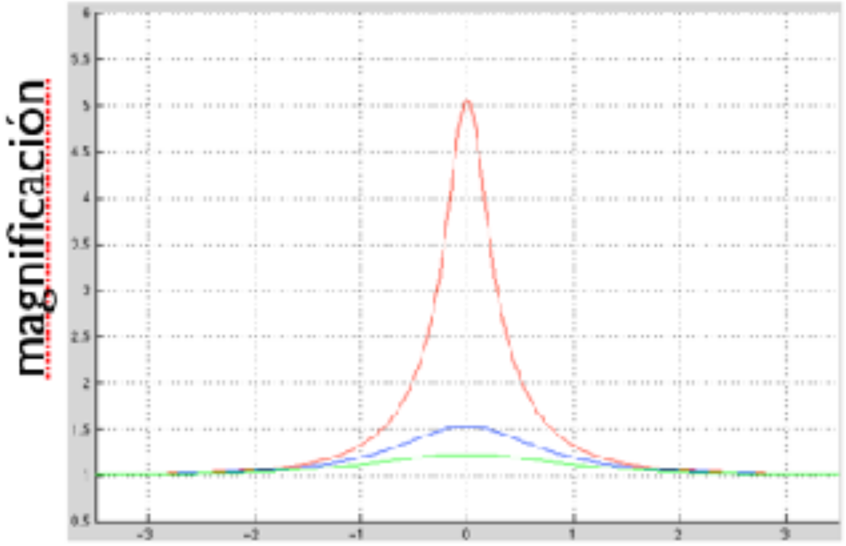
$$\theta_E = \sqrt{\frac{D_{LS}}{D_{OS}D_{OL}}\frac{4GM}{c^2}}$$

- Light magnification of a star produced by the strong lensing effect of a closer condensed object

$\omega_\perp$: velocidad angular relativa entre la fuente y la lente

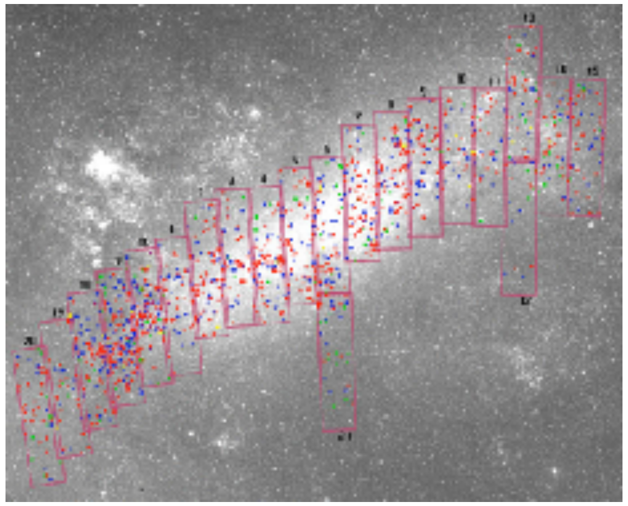


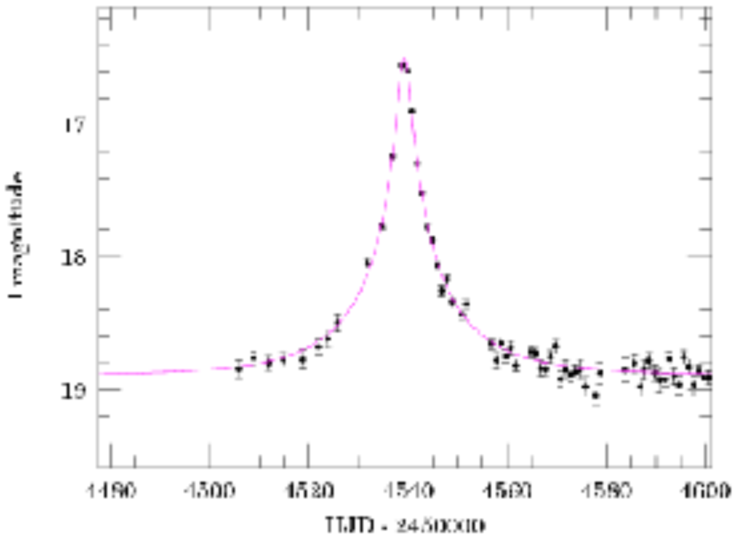
Plano de las fuentes



Tiempo en unidades del tiempo de Einstein

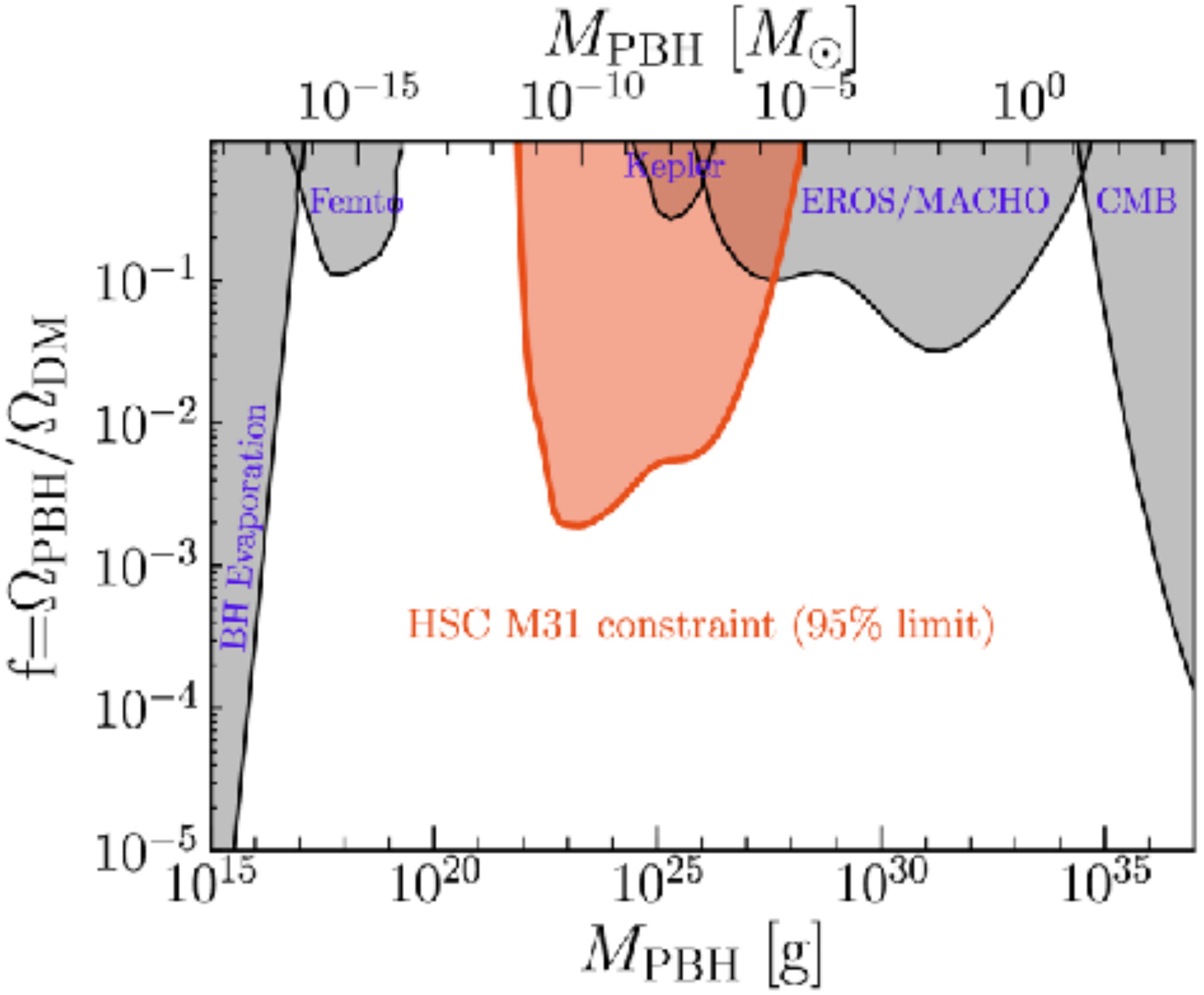
- Relative motion causes a variation in the magnification
- Need to monitor a large number of stars (Einstein though this effect was undetectable)





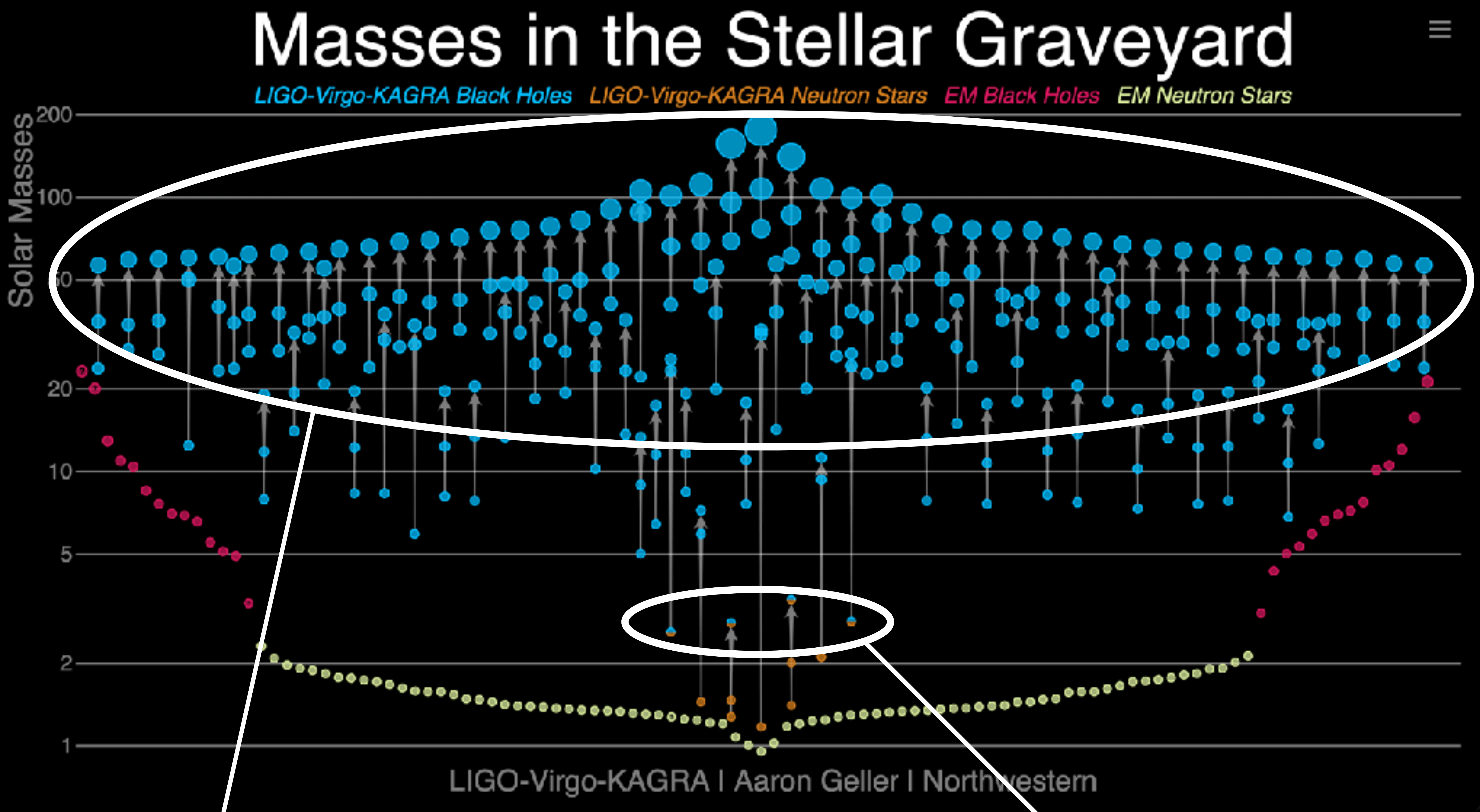
# What can we study with *microlensing*?

- Mass census in the galaxy
- Detection (or not) of microlensing events sets constraints on the abundance of objects of a given mass (or mass spectrum)
- Dark Matter candidates: typically Primordial Black Holes
- No sign of dark condensed objects compatible with dark matter abundance
- Wide mass range discarded by microlensing surveys
- Still a low-mass window unexplored (finite source and wave optics effects, cadence, blending, femtolensing)
- Lensing by "exotic" DM candidates (axion mini-clusters, boson stars, etc.) Impact on event rates and *light curves*



arXiv:1701.02151v3

https://ligo.northwestern.edu/media/mass-plot/index.html

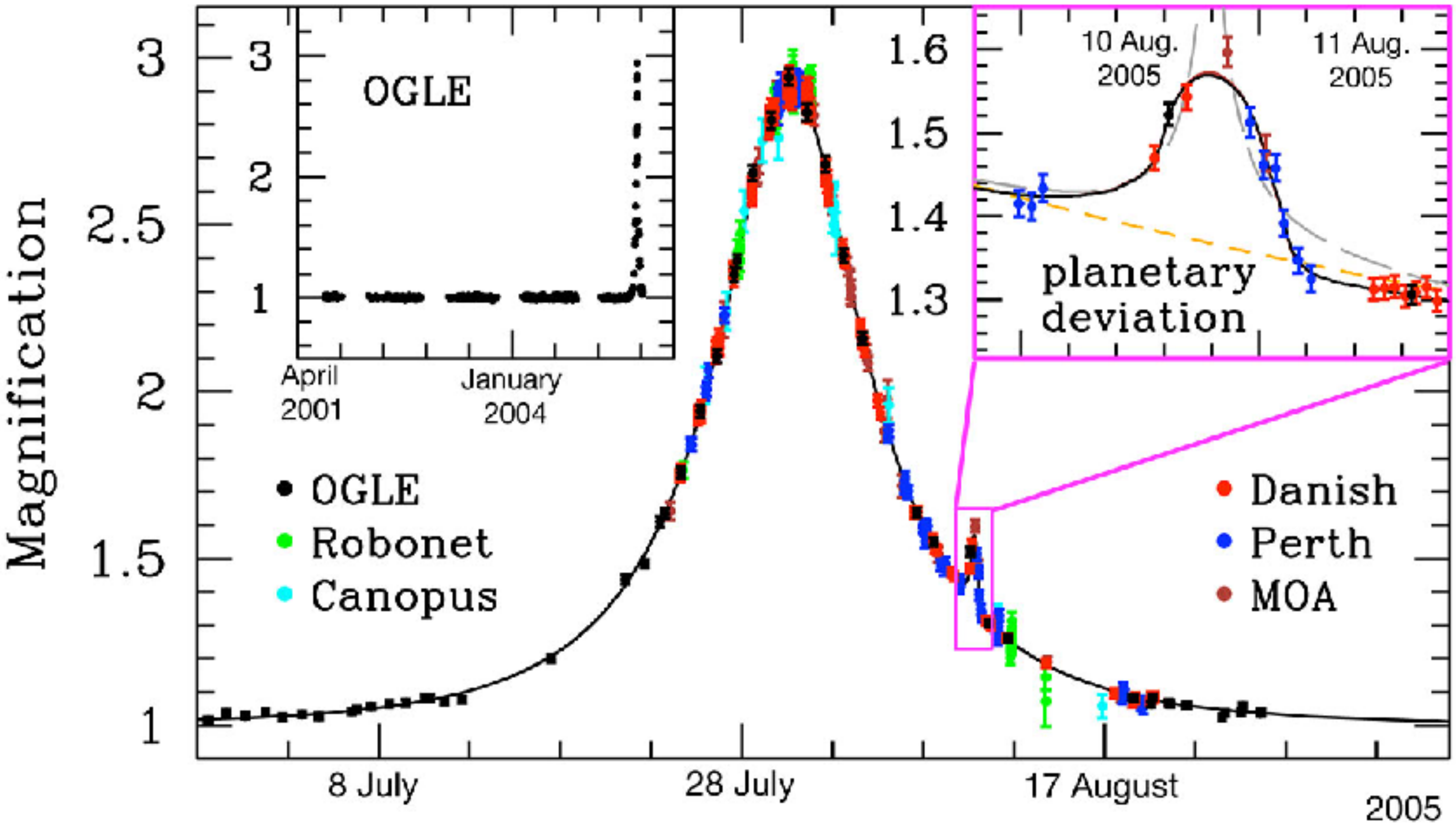


Can we spot those high-mass Black-Holes by other means?
What is the population of rogue BH?

Are those really mass-gap objects?
Or they are lensed?
Can we find such objects by other means?

**Microlensing can help us build a more complete census of compact objects**

# Exoplanets



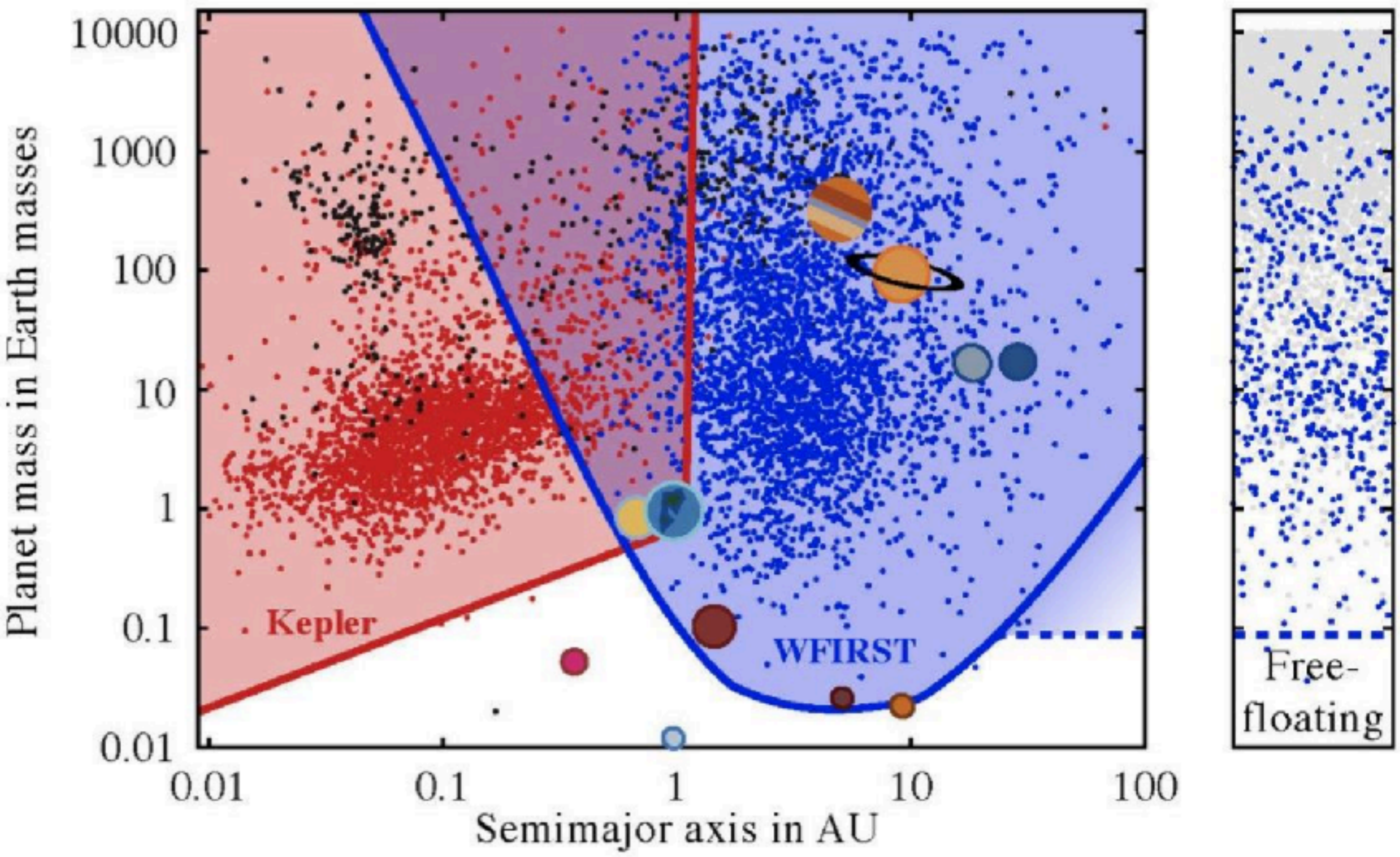
https://www.eso.org/public/news/eso0603/

http://exoplanet.eu/catalog/

- 240 extra-solar planets discovered so far (> 50 in 2022!)
- Typical “planet anomalies”
- Require much higher cadence

# Exoplanet discoveries

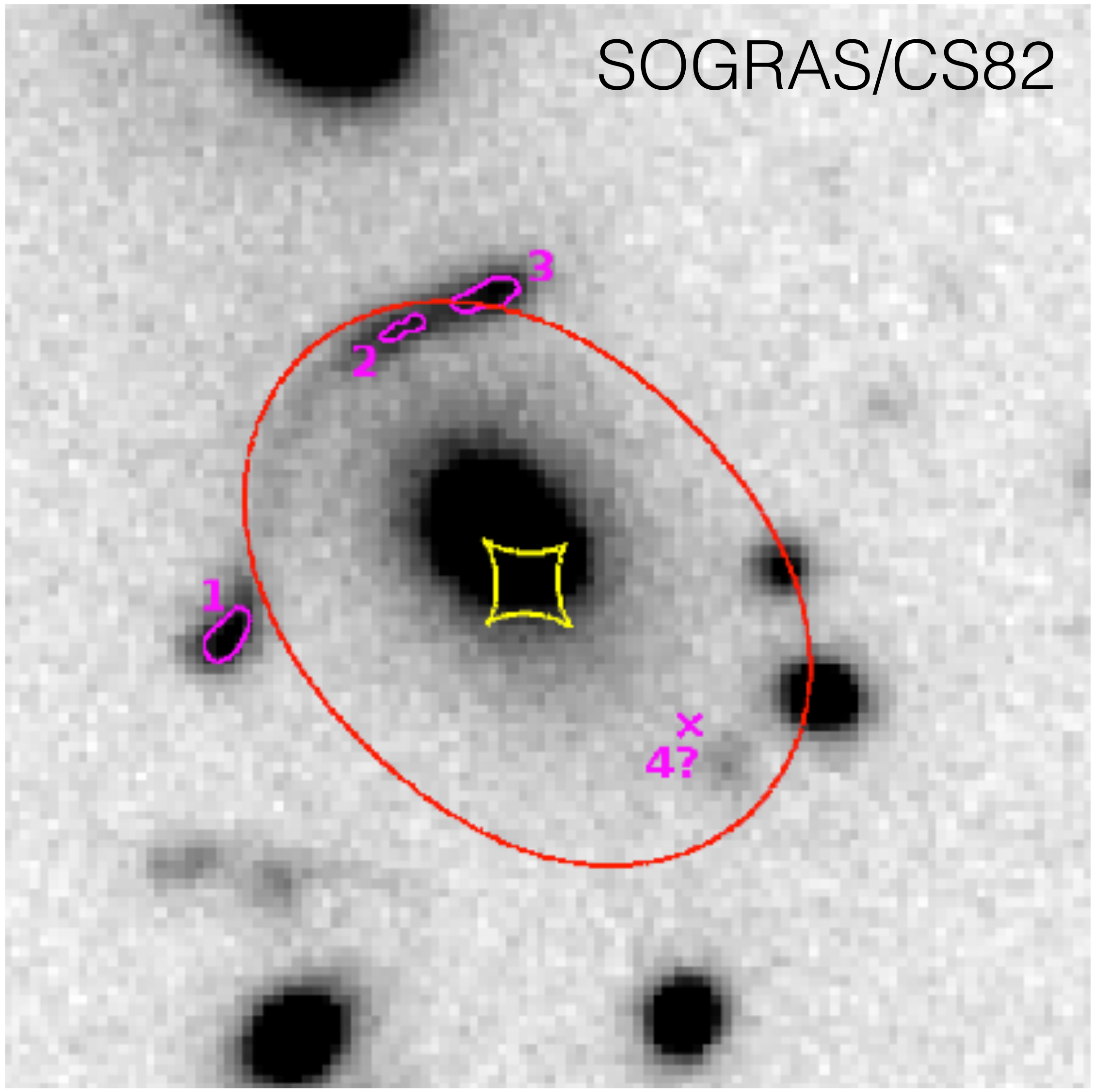
Roman: 2.4 m space-telescope, FoV 0.8 sq-deg



# Strong Lensing of extended lenses

Mass reconstruction from multiple images

# Inverse Modeling: Mapping the Mass



Use systems of multiple images to determine the lensing potential

$$\chi^2_{\mathrm{lente}} := \sum_i \left( \frac{\vec{\theta}_i^{\,\mathrm{obs}} - \vec{\theta}^{\,\mathrm{mod}}\left(\vec{\beta}, \vec{\Pi}\right)}{\sigma_i^{\mathrm{obs}}} \right)^2$$

Multiple image positions

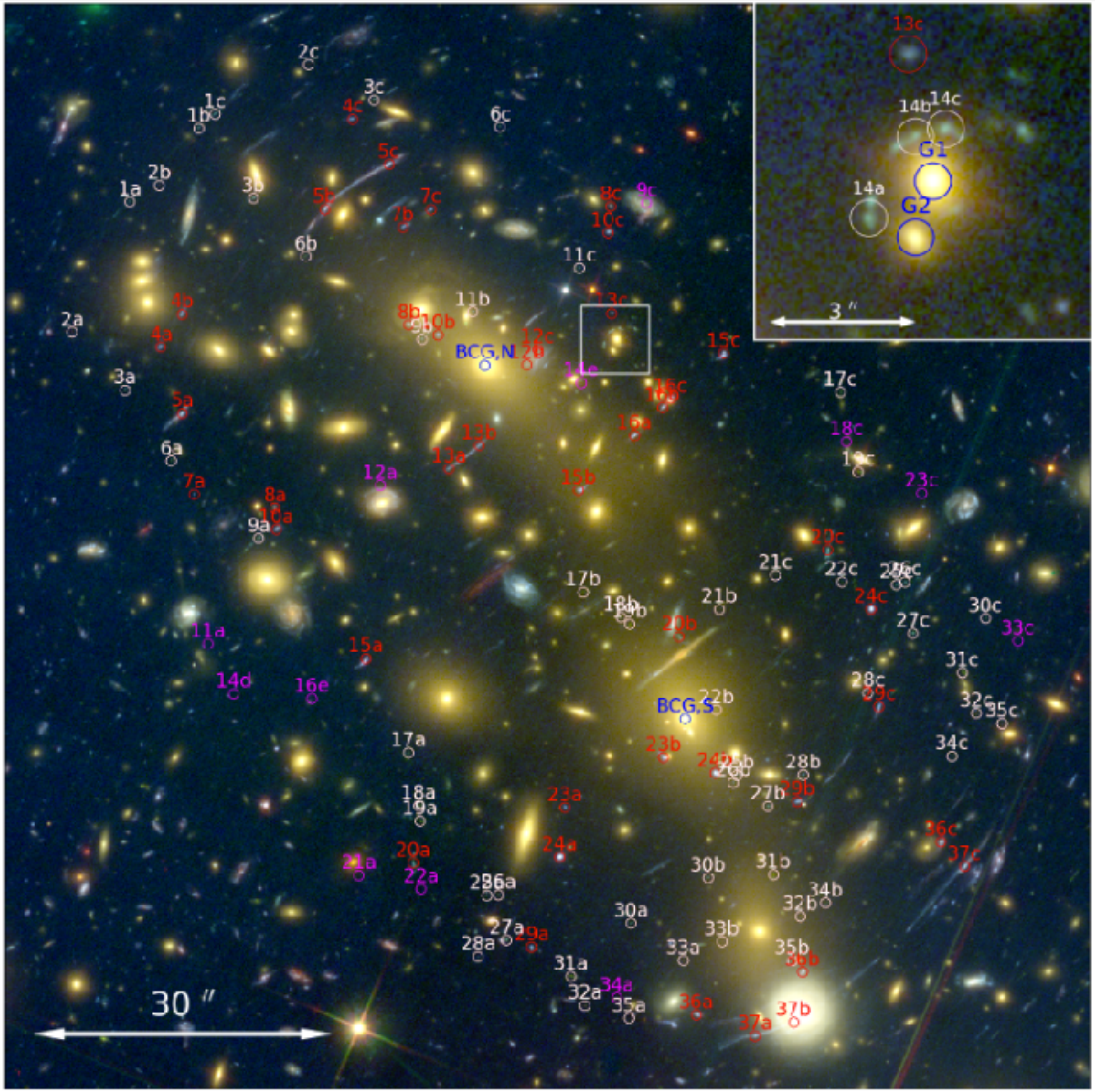
Error on image positions

$\vec{\Pi}$: parameters of the mass distribution and cosmological parameters

Can use the whole light distribution of the images to improve the lens mass reconstruction and reconstruct the source

The more multiple images, the more constrains: Cluster x Galaxy scales

# Example MACS J0416.1-2403



Caminha et al. 2016

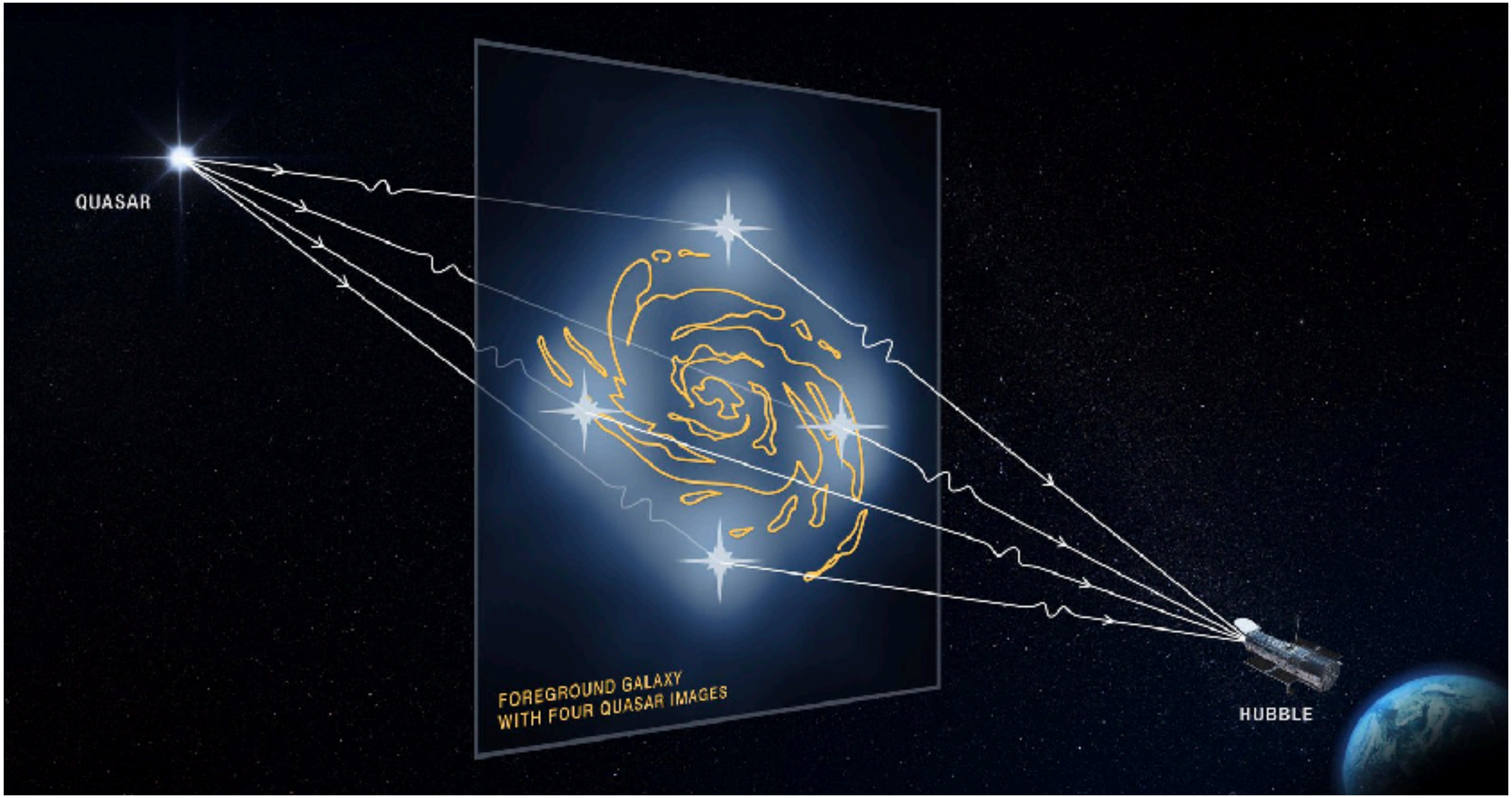
- HST + MUSE IFU
- 102 multiple images!
- Discovery of new systems in the data cube
- Robust determination of projected mass
- Cosmological constraints

# Time Delay

# Time delay

Geometric + gravitational redshift + Doppler (observer frame)

$$\delta t_L = \delta t_{\rm geom} + \delta t_{\rm grav}$$



https://www.jpl.nasa.gov/images/pia23641-gravitational-lensing-graphic

# Time delay

Geometric + gravitational redshift + Doppler (observer frame)

$$\delta t_L = \delta t_{\rm geom} + \delta t_{\rm grav}$$

$$\delta t_{\rm geom} = \delta L/c = \frac{D_{OS} D_{OL}}{2 c D_{LS}} (\vec{\theta} - \vec{\beta})^2 \qquad \delta t_{grav} = -\frac{2}{c^3}\psi$$

$$\delta_{t_O}/\delta_{t_L} = a_O/a_L = (1 + z_L) \qquad \Psi \equiv \frac{2}{c^2}\frac{D_{\rm LS}}{D_{\rm OS} D_{\rm OL}}\psi$$

$$\boxed{\delta t = (1 + z_L)\frac{D_{OS} D_{OL}}{c D_{LS}} \left(\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \Psi\right)}$$

$\vec{\nabla}_\theta(\delta t) = 0$ ⇒ **Lens equation!**

Obs.: rigorous derivation at Petters, Levine, Wambsganss

# Time Delay

$$\Delta t_{ij} = \frac{D_\Delta}{c}\left(\frac{1}{2}(\vec{\theta_i} - \vec{\beta})^2 - \Psi(\theta_i) - \frac{1}{2}(\vec{\theta_j} - \vec{\beta})^2 + \Psi(\theta_j)\right)$$

inverse modeling
(multiple images)

Where $D_\Delta = (1 + z_L)\frac{D_{OS}D_{OL}}{D_{LS}}$

“time delay distance”

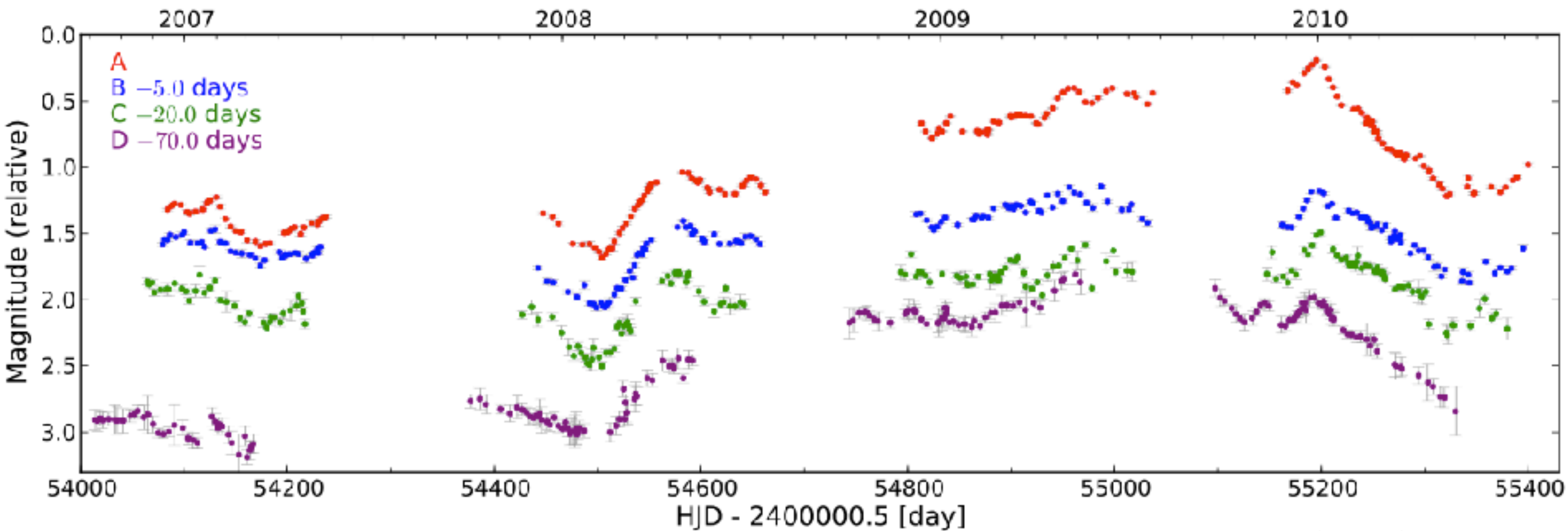
Main cosmological dependence $\propto H_0^{-1}$

# Quasar light-curves

Time delay between images

$$\Delta t_{ij} = \delta t\left(\vec{\theta}_i, \vec{\beta}\right) - \delta t\left(\vec{\theta}_j, \vec{\beta}\right)$$

$$= (1+z_L)\frac{D_{OS}D_{OL}}{cD_{LS}}\left(\frac{1}{2}(\vec{\theta_i} - \vec{\beta})^2 - \Psi(\theta_i) - \frac{1}{2}(\vec{\theta_j} - \vec{\beta})^2 + \Psi(\theta_j)\right)$$

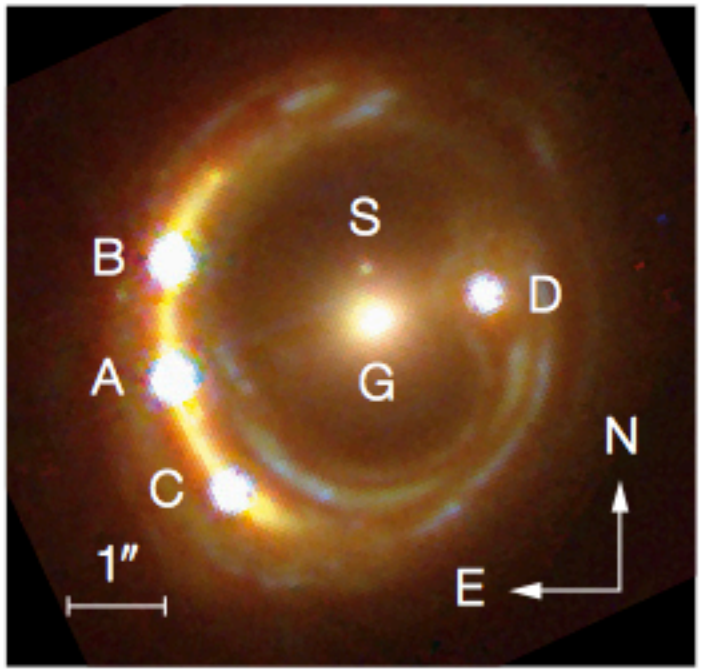


Tewes et al.: COSMOGRAIL XI

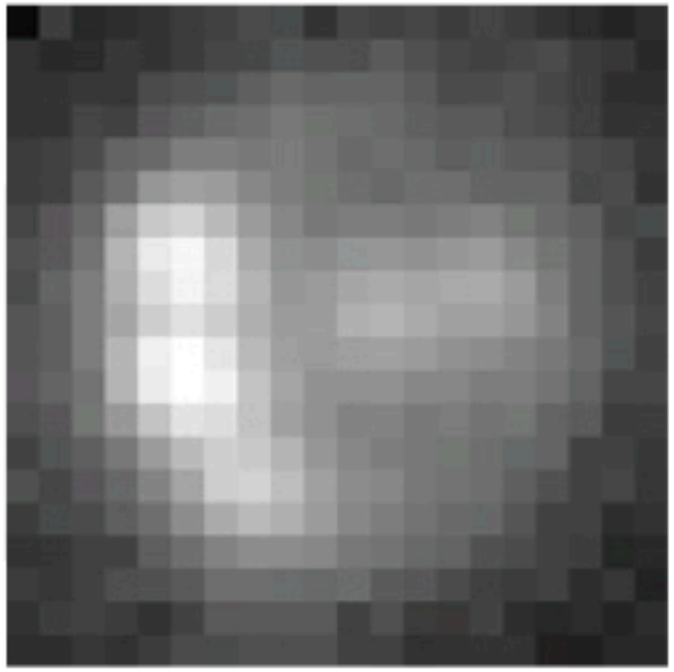
# Exemple: QSO RX J1131−1231

- COSMOGRAIL: the COSmological MOnitoring of GRAvItational Lenses
- Light-curves + lens model (+“all the rest”)

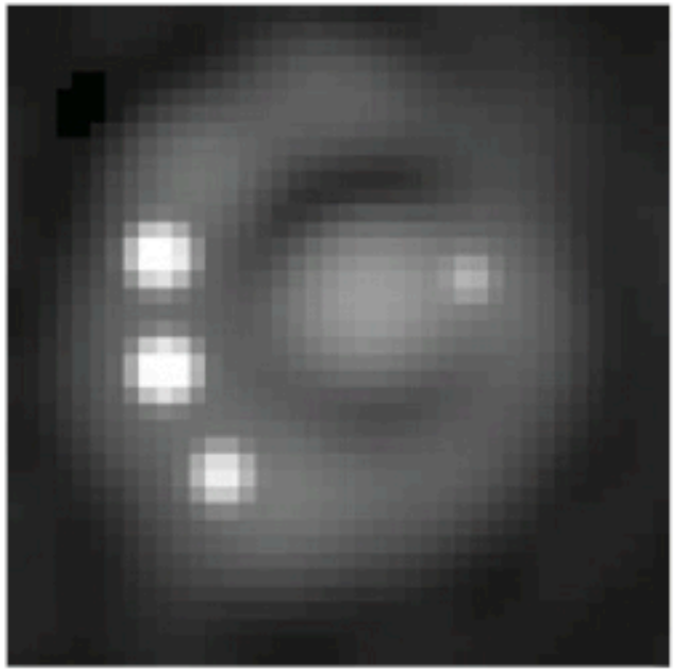
$$\kappa_{\mathrm{pl}}(\theta_1, \theta_2) = \frac{3-\gamma'}{2}\left(\frac{\theta_{\mathrm{E}}}{\sqrt{q\theta_1^2 + \theta_2^2/q}}\right)^{\gamma'-1}$$



Hubble Space Telescope



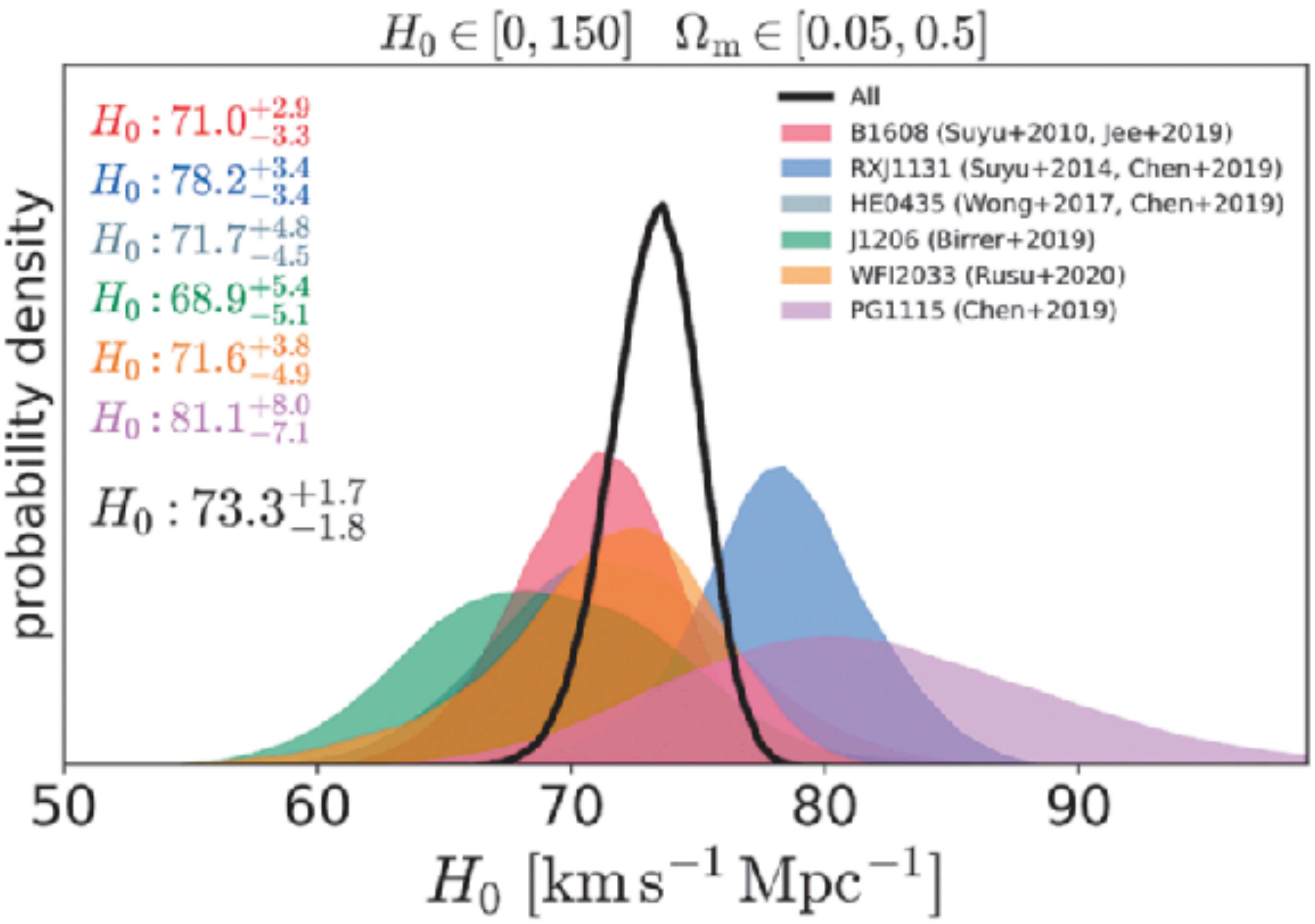
Swiss Leonhard Euler Telescope



Euler deconvolved

# Exemple: QSO RX J1131−1231

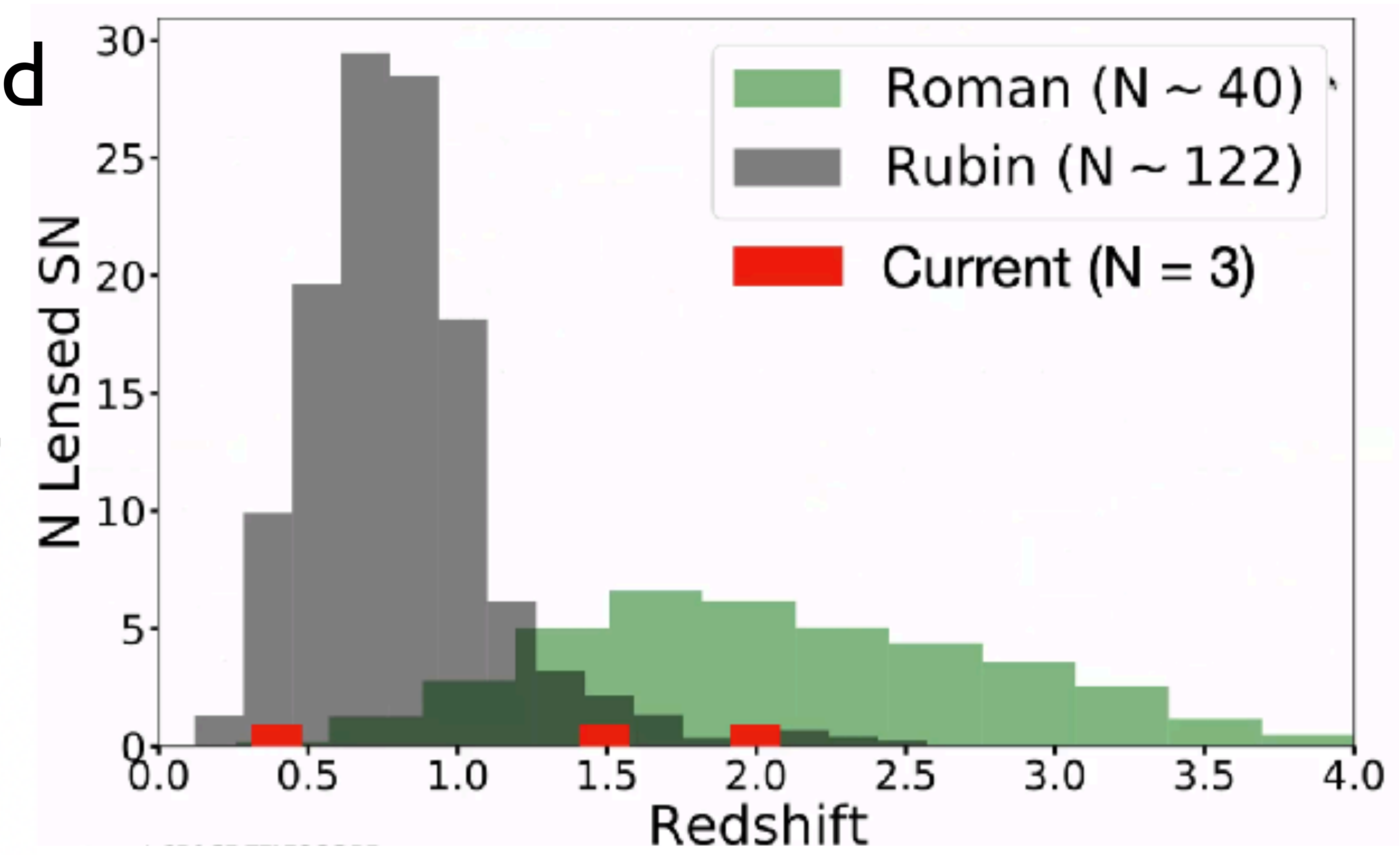
- COSMOGRAIL: the COSmological MOnitoring of GRAvItational Lenses
- Example: contraints on $H_0$

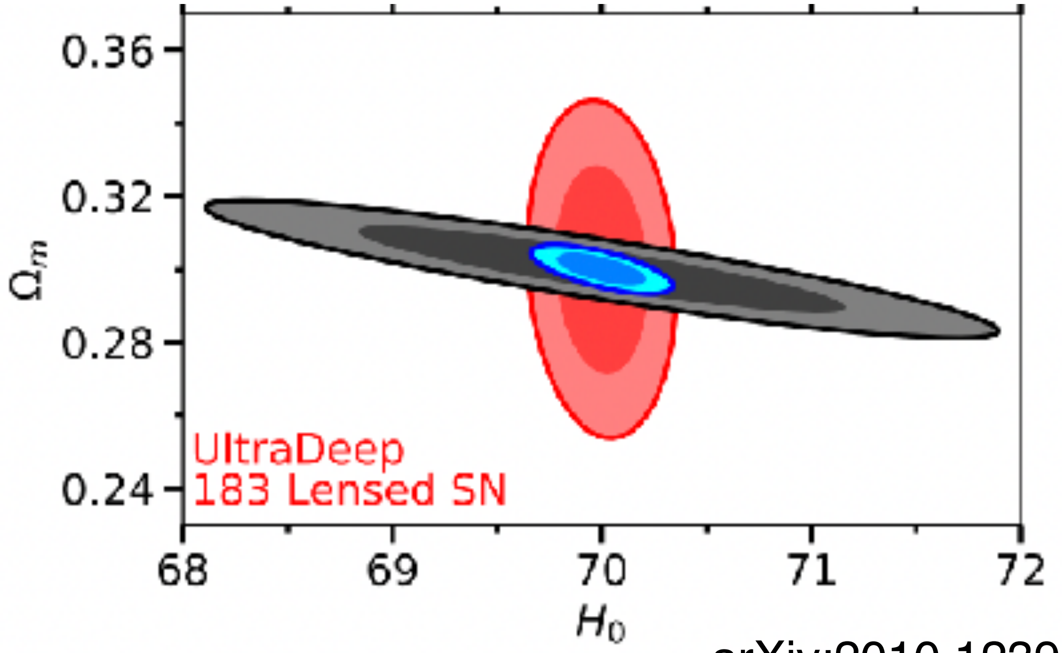


https://www.asj.or.jp/jp/activities/geppou/item/115-11_705en.pdf

# Lensing of Supernovae

- Power of Standard candles + time-delays + Strong Lensing Modeling
- Emerging field
- MMA sources…





arXiv:2010.12399

# Diffractive Gravitational Lensing

# Wave optics effects

- If the wavelength is comparable to the Schwarzschild radius, one has to account for wave optics!
- Maxwell's equations on a curved background
Solution for the amplitude ratio of the field:

$$F(\omega, \vec{\eta}) = \frac{D_{\mathrm{S}}}{D_{\mathrm{L}} D_{\mathrm{LS}}} \frac{\omega}{2\pi i} \int d^2\xi \exp\left[i\omega t'(\vec{\xi}, \vec{\eta})\right]$$

Where *t* is the time delay function

and the dimensionless, characteristic frequency is

$$w = \frac{4GM}{c^2}\omega(1 + z_L) = 4\pi(1 + z_{\mathrm{L}})\frac{r_{\mathrm{sch}}(M)}{\lambda}$$

# Wave optics effects

- If the wavelength is comparable to the Schwarzschild radius, one has to account for wave optics!
- Maxwell’s equations on a curved background
Solution for a point lens

$$F(w,u) = e^{\frac{i}{2}\left(u^2-\ln\left(w/2\right)\right)} e^{\frac{\pi}{4}w} \Gamma\left(1-\frac{i}{2}w\right) {}_1F_1\left(1-\frac{i}{2}w, 1; -\frac{i}{2}wu^2\right)$$

The magnification is therefore:

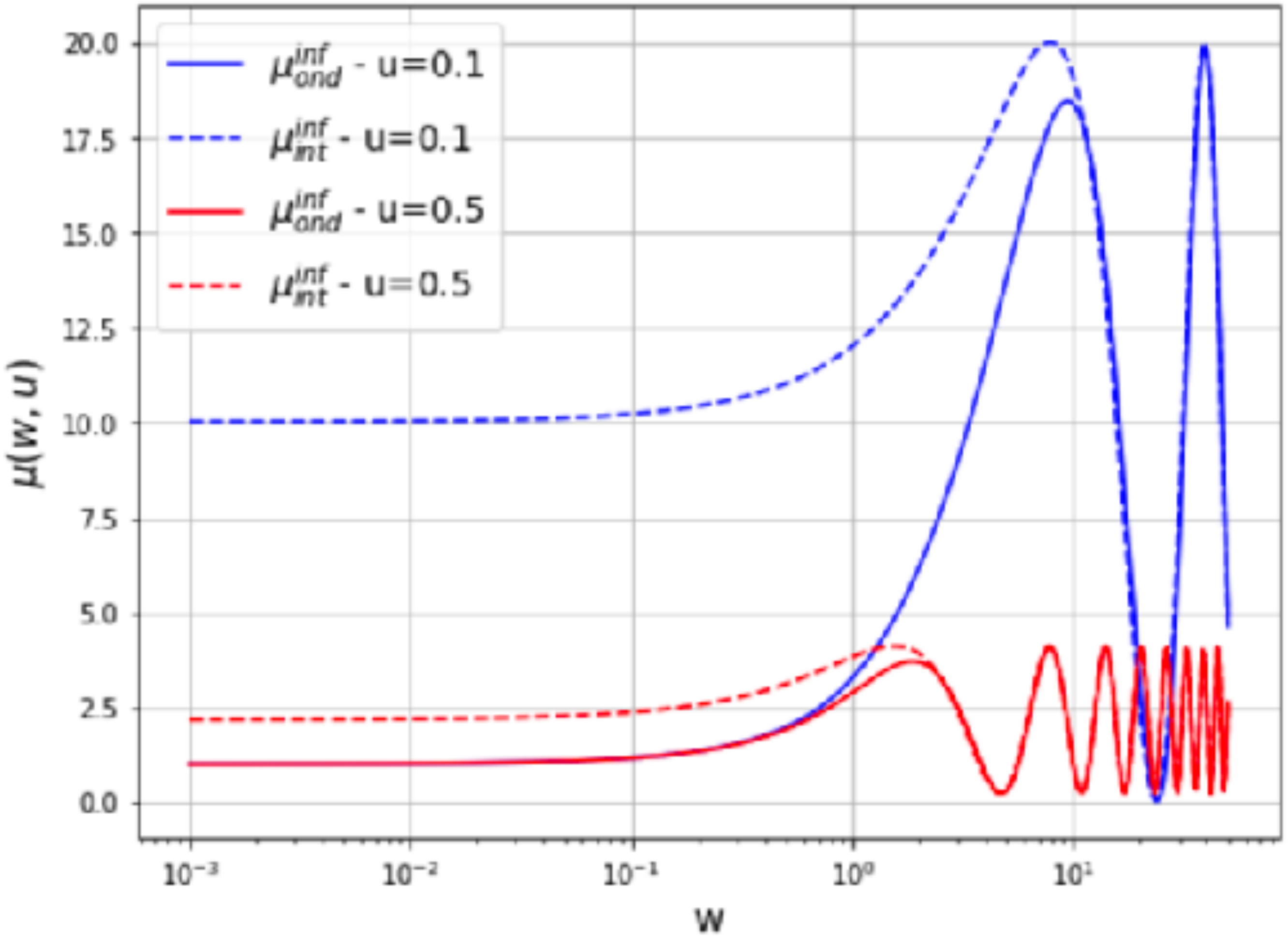
$$\mu^{\mathrm{inf}}_{\mathrm{ond}}(w,u) = \frac{\pi w}{1-e^{\pi w}} \left| {}_1F_1\left(\frac{i}{2}w, 1; \frac{i}{2}wu^2\right)\right|^2$$

$$w = \frac{4GM}{c^2}\omega(1+z_L) = 4\pi(1+z_{\mathrm{L}})\frac{r_{\mathrm{sch}}(M)}{\lambda}$$

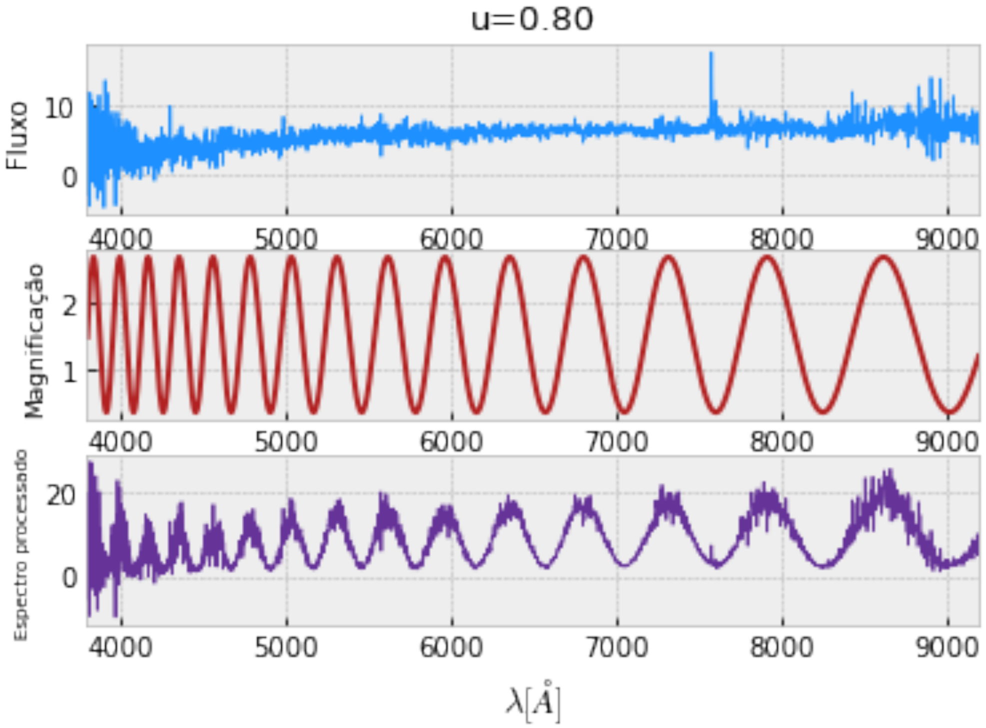
# Wave optics effects

- For high frequencies

$$\mu_{\text{int}}^{\text{inf}}(w,u) = \frac{u^2+2}{u\sqrt{u^2+4}} + \frac{2}{u\sqrt{u^2+4}}\sin\left\{w\left[\frac{1}{2}u\sqrt{u^2+4}+\ln\left(\frac{\sqrt{u^2+4}+u}{u-\sqrt{u^2+4}}\right)\right]\right\}$$



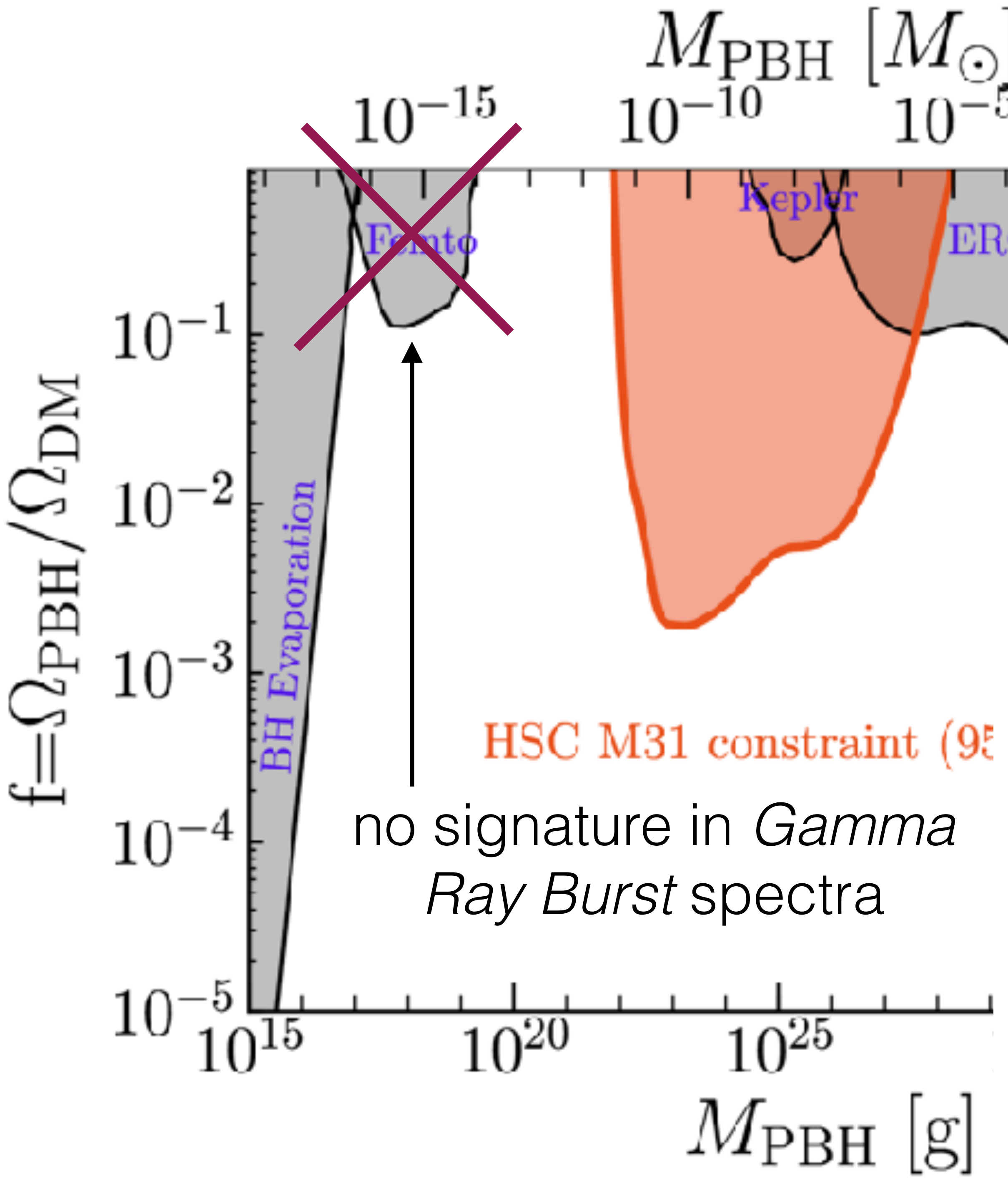
Effect on a spectrum



Femtolensing!

$$w = \frac{4GM}{c^2}\omega(1+z_L) = 4\pi(1+z_{\rm L})\frac{r_{\rm sch}(M)}{\lambda}$$

# Femtolensing: wave optics



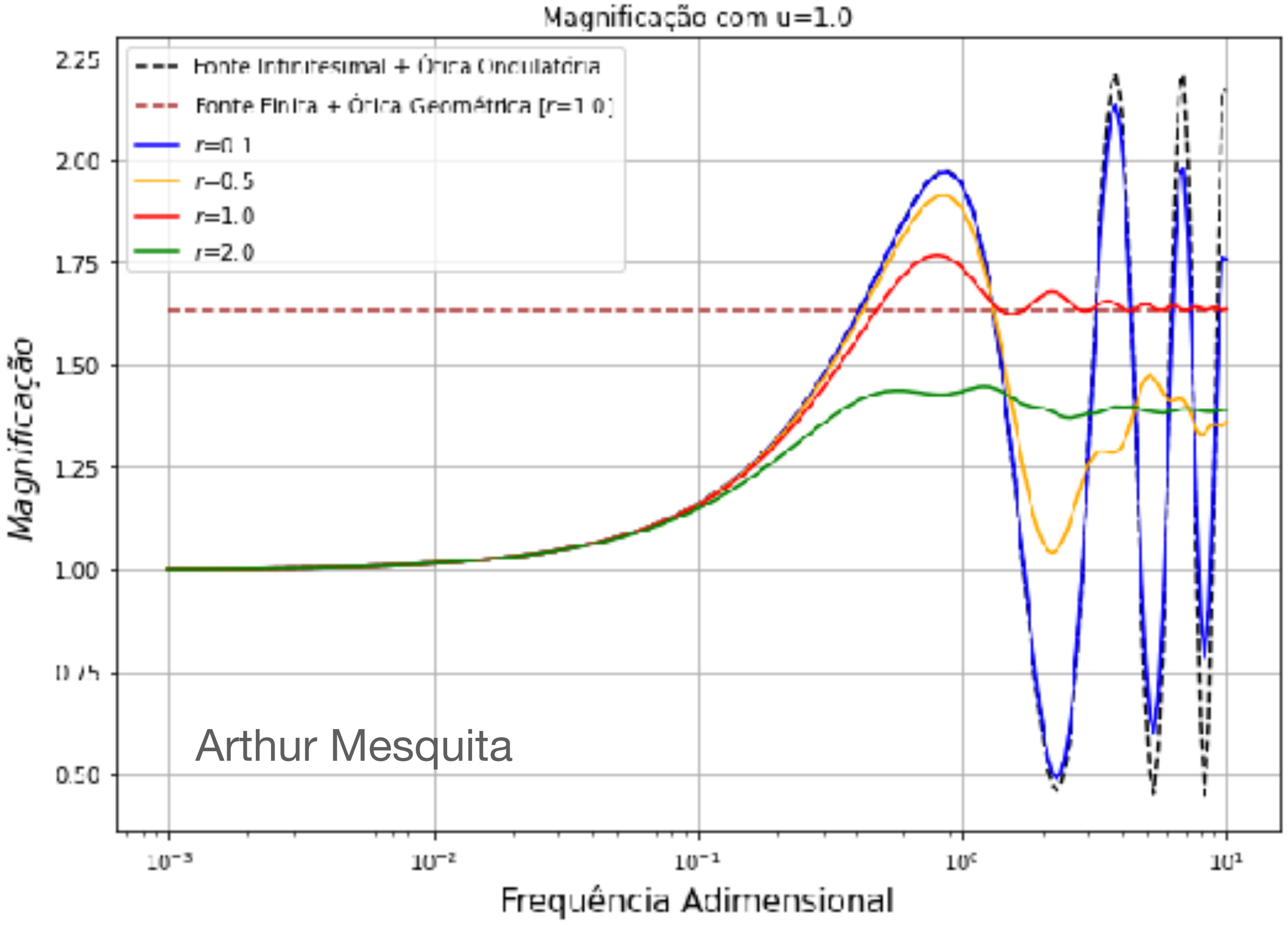
no signature in *Gamma Ray Burst* spectra

arXiv:1701.02151v3

Finite size effects destroy the signal!

arXiv:1807.11495

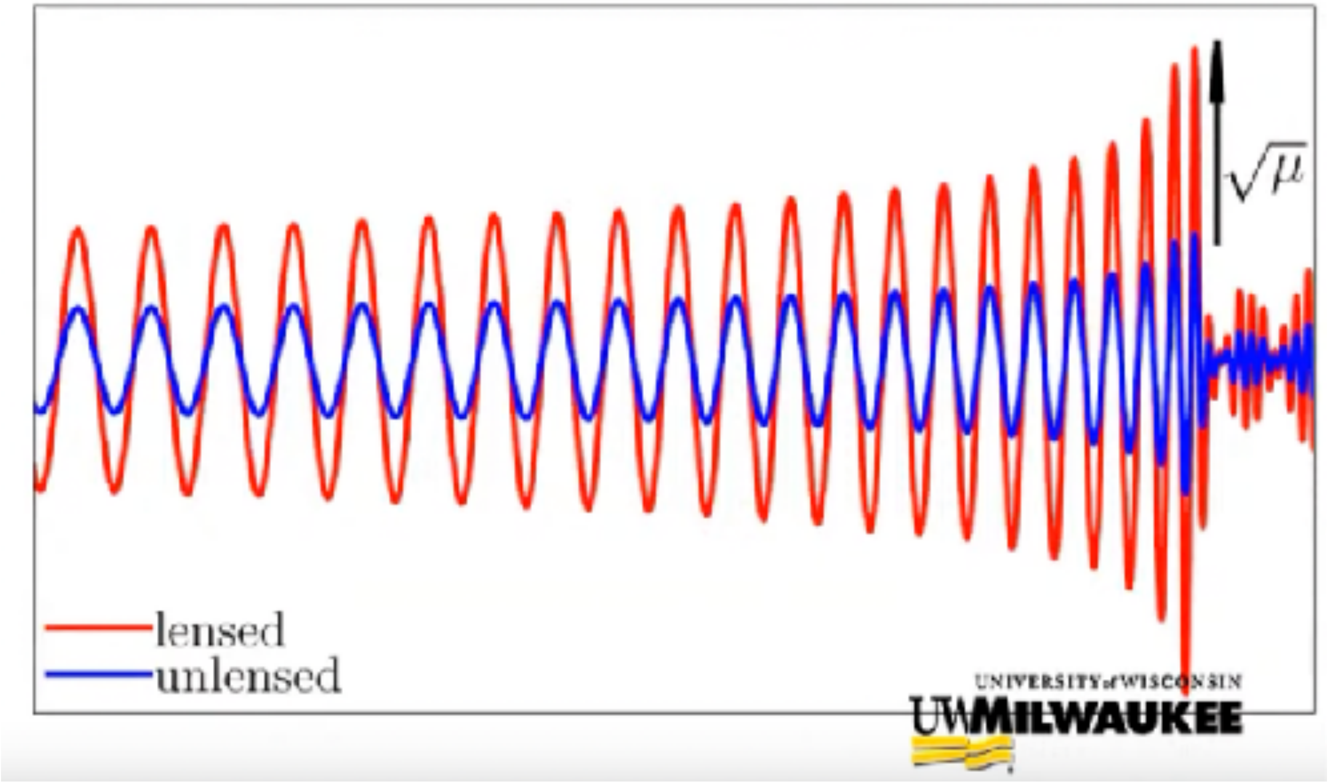
Projections for femtolensing of GRB: arXiv:1807.11495



# Lensing of Gravitational Waves

# Lensing of Gravitational Waves

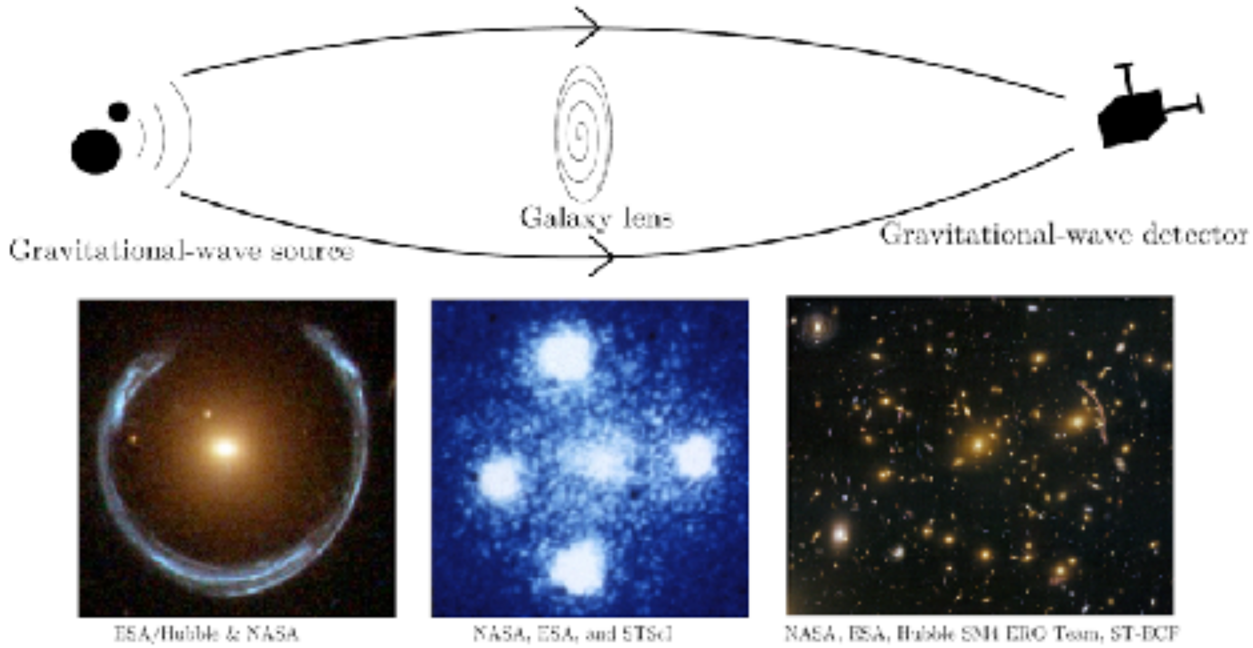
- GW are tensor waves but strain (amplitude) follows same lensing equations
- Detected in interferometers
- Regimes of GW lensing
    - **Magnification:** statistical or individual



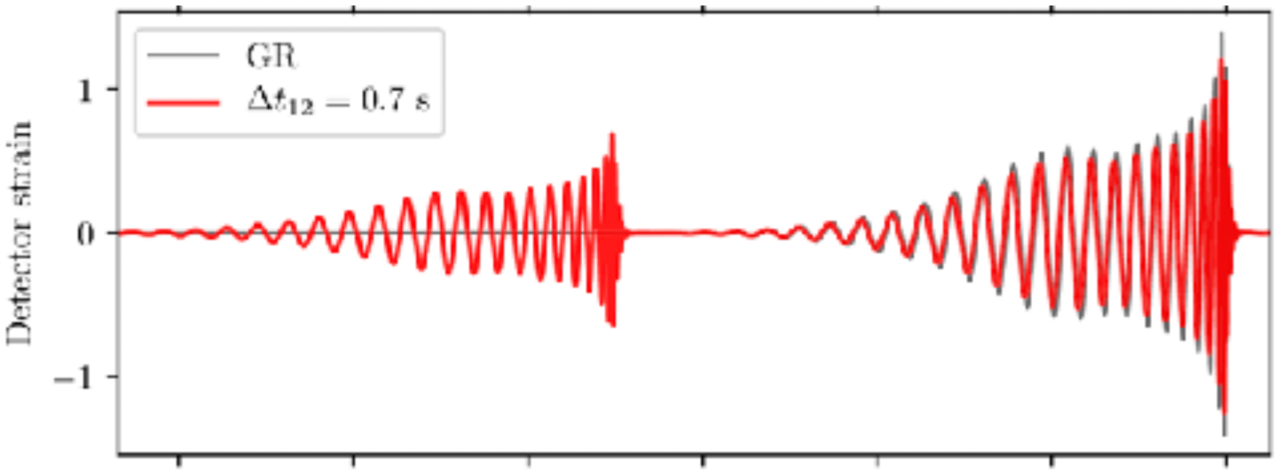
    - could explain mass gap events?

# Regimes of GW lensing

- Magnification: statistical or individual
  - could explain mass gap events?
- Strong Lensing (lens is galaxy or cluster): multiple images
  - different arrival times and different magnifications



https://www.ligo.org/science/Publication-O3aLensing/images/O3aLensing-Fig1.png



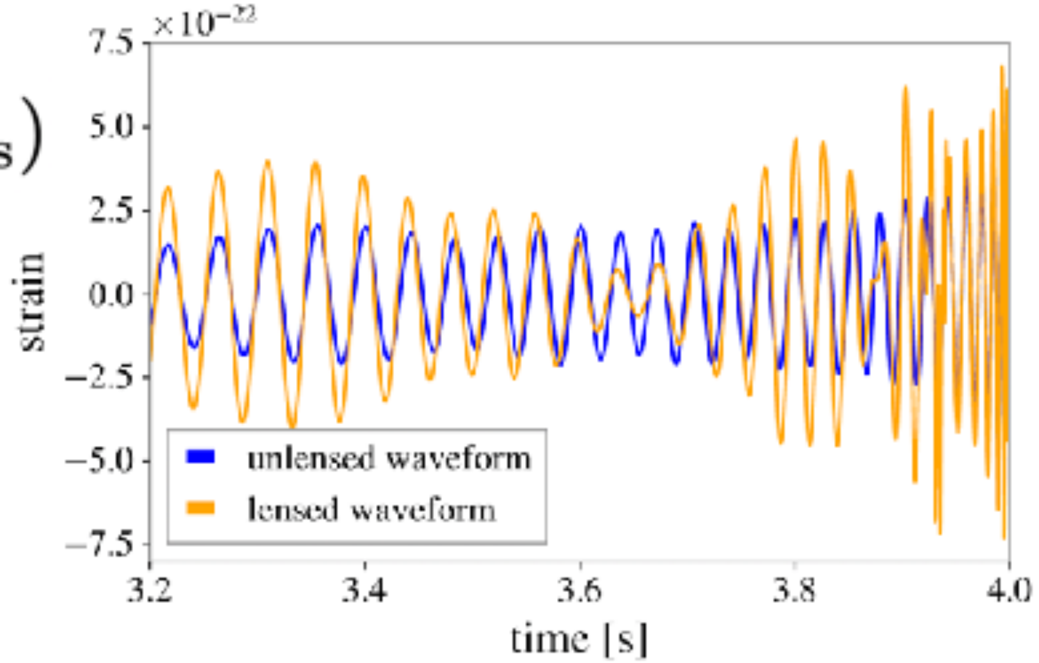
Ezquiaga & Zumalacárregui, PRD 102, 124048 (2020)

typical time delays from minutes to months

# Regimes of GW lensing

- Magnification: statistical or individual
  - could explain mass gap events?
- Strong Lensing (lens is galaxy or cluster): multiple images
  - different arrival times and different magnifications
- Microlensing (lens is a massive BH):
  - frequency dependent magnification: beating pattern

$$h_{\rm ML}(f,\theta_{\rm s},\theta_{\rm ML}) = F(f, M^z_{\rm ML}, y) \times h_{\rm U} \quad (f,\theta_{\rm s})$$

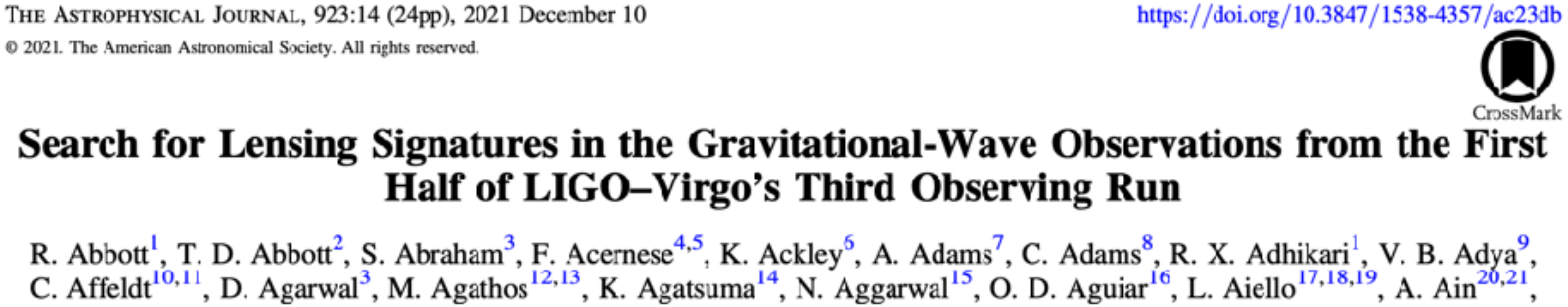


arXiv:2110.03308

# Has lensing of GW been observed?

- Methods to detect lensing effect on the GW signal have been developed and searches have been conducted on the existing data (O3)

# Has lensing of GW been observed?

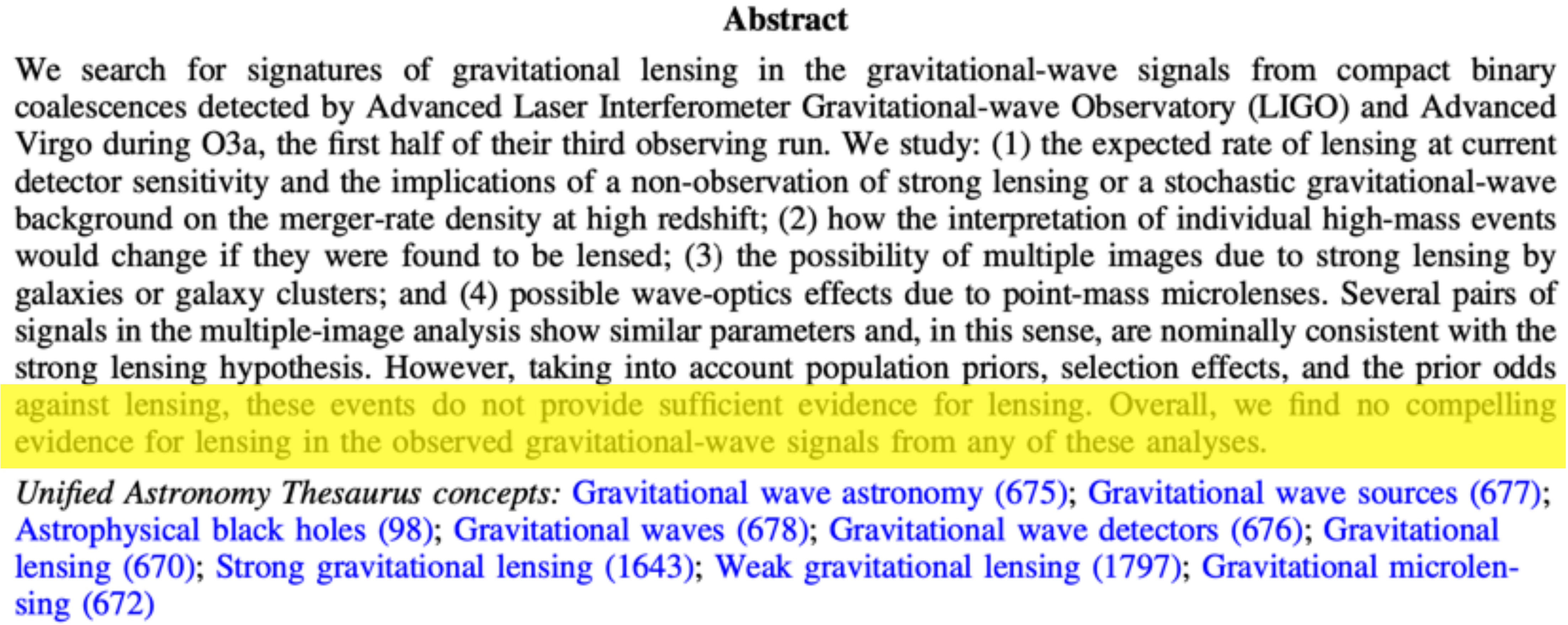


The Astrophysical Journal, 923:14 (24pp), 2021 December 10
https://doi.org/10.3847/1538-4357/ac23db

## Search for Lensing Signatures in the Gravitational-Wave Observations from the First Half of LIGO–Virgo's Third Observing Run

R. Abbott[1], T. D. Abbott[2], S. Abraham[3], F. Acernese[4,5], K. Ackley[6], A. Adams[7], C. Adams[8], R. X. Adhikari[1], V. B. Adya[9], C. Affeldt[10,11], D. Agarwal[3], M. Agathos[12,13], K. Agatsuma[14], N. Aggarwal[15], O. D. Aguiar[16], L. Aiello[17,18,19], A. Ain[20,21],

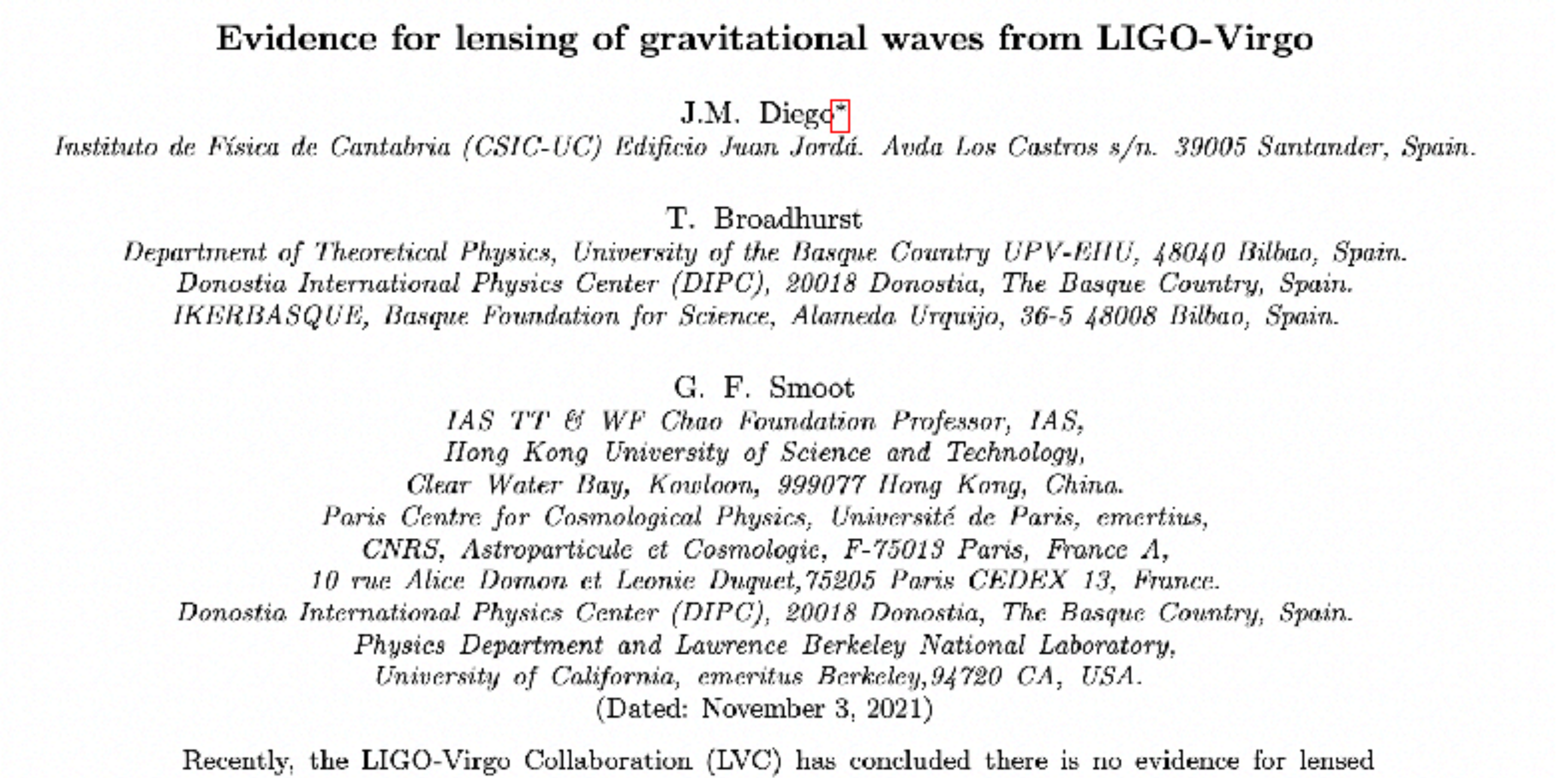
[7 pages of names + addresses]



### Abstract

We search for signatures of gravitational lensing in the gravitational-wave signals from compact binary coalescences detected by Advanced Laser Interferometer Gravitational-wave Observatory (LIGO) and Advanced Virgo during O3a, the first half of their third observing run. We study: (1) the expected rate of lensing at current detector sensitivity and the implications of a non-observation of strong lensing or a stochastic gravitational-wave background on the merger-rate density at high redshift; (2) how the interpretation of individual high-mass events would change if they were found to be lensed; (3) the possibility of multiple images due to strong lensing by galaxies or galaxy clusters; and (4) possible wave-optics effects due to point-mass microlenses. Several pairs of signals in the multiple-image analysis show similar parameters and, in this sense, are nominally consistent with the strong lensing hypothesis. However, taking into account population priors, selection effects, and the prior odds against lensing, these events do not provide sufficient evidence for lensing. Overall, we find no compelling evidence for lensing in the observed gravitational-wave signals from any of these analyses.

*Unified Astronomy Thesaurus concepts:* Gravitational wave astronomy (675); Gravitational wave sources (677); Astrophysical black holes (98); Gravitational waves (678); Gravitational wave detectors (676); Gravitational lensing (670); Strong gravitational lensing (1643); Weak gravitational lensing (1797); Gravitational microlensing (672)



# Evidence for lensing of gravitational waves from LIGO-Virgo

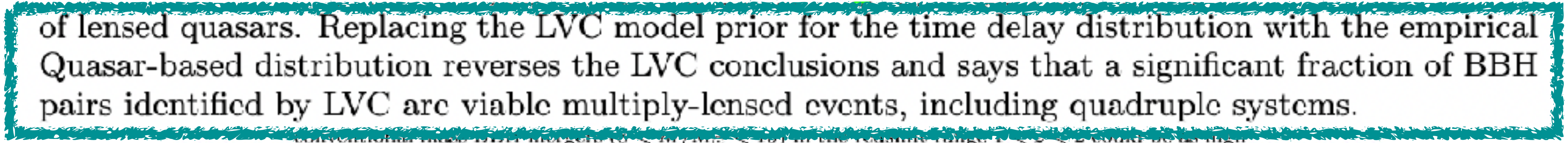
J.M. Diego[*]
*Instituto de Física de Cantabria (CSIC-UC) Edificio Juan Jordá. Avda Los Castros s/n. 39005 Santander, Spain.*

T. Broadhurst
*Department of Theoretical Physics, University of the Basque Country UPV-EHU, 48040 Bilbao, Spain.*
*Donostia International Physics Center (DIPC), 20018 Donostia, The Basque Country, Spain.*
*IKERBASQUE, Basque Foundation for Science, Alameda Urquijo, 36-5 48008 Bilbao, Spain.*

G. F. Smoot
*IAS TT & WF Chao Foundation Professor, IAS,*
*Hong Kong University of Science and Technology,*
*Clear Water Bay, Kowloon, 999077 Hong Kong, China.*
*Paris Centre for Cosmological Physics, Université de Paris, emertius,*
*CNRS, Astroparticule et Cosmologie, F-75013 Paris, France A,*
*10 rue Alice Domon et Leonie Duquet,75205 Paris CEDEX 13, France.*
*Donostia International Physics Center (DIPC), 20018 Donostia, The Basque Country, Spain.*
*Physics Department and Lawrence Berkeley National Laboratory,*
*University of California, emeritus Berkeley,94720 CA, USA.*

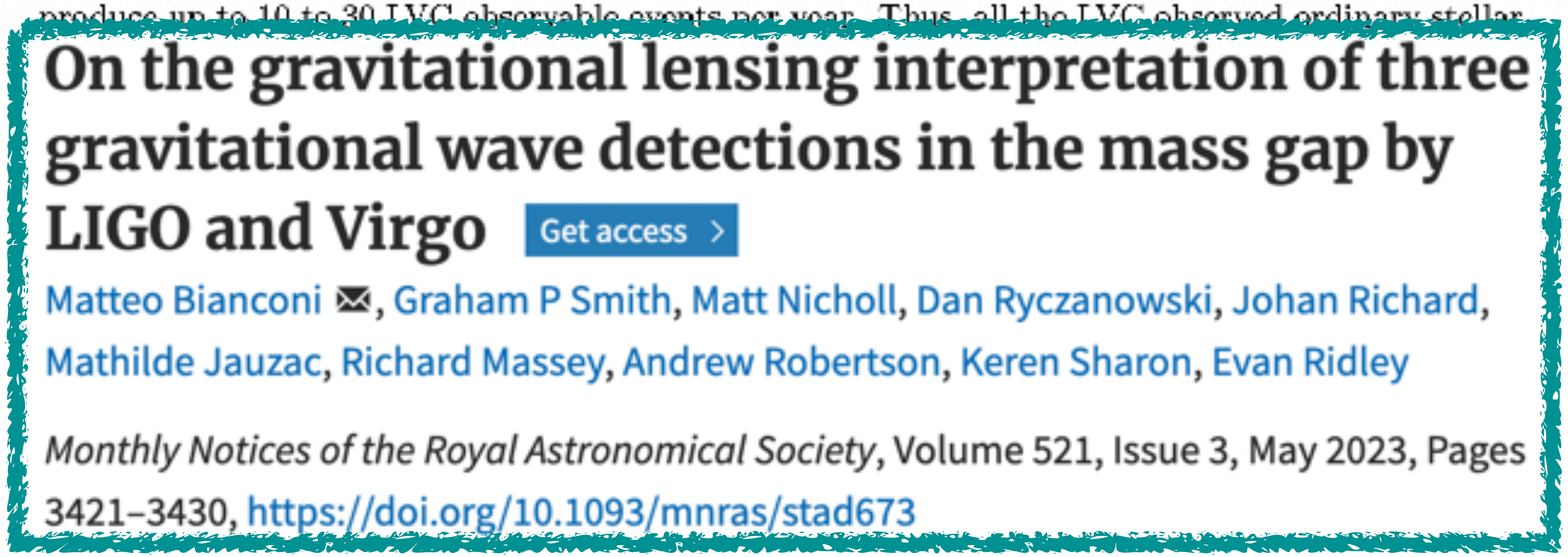
(Dated: November 3, 2021)

Recently, the LIGO-Virgo Collaboration (LVC) has concluded there is no evidence for lensed



of lensed quasars. Replacing the LVC model prior for the time delay distribution with the empirical Quasar-based distribution reverses the LVC conclusions and says that a significant fraction of BBH pairs identified by LVC are viable multiply-lensed events, including quadruple systems.

as $O(10^7)$ events per year, more than sufficient to compensate for the intrinsically low probability of lensing. To reach the LVC trigger threshold these events require high magnification, but would still produce up to 10 to 30 LVC observable events per year. Thus, all the LVC observed ordinary stellar



# On the gravitational lensing interpretation of three gravitational wave detections in the mass gap by LIGO and Virgo

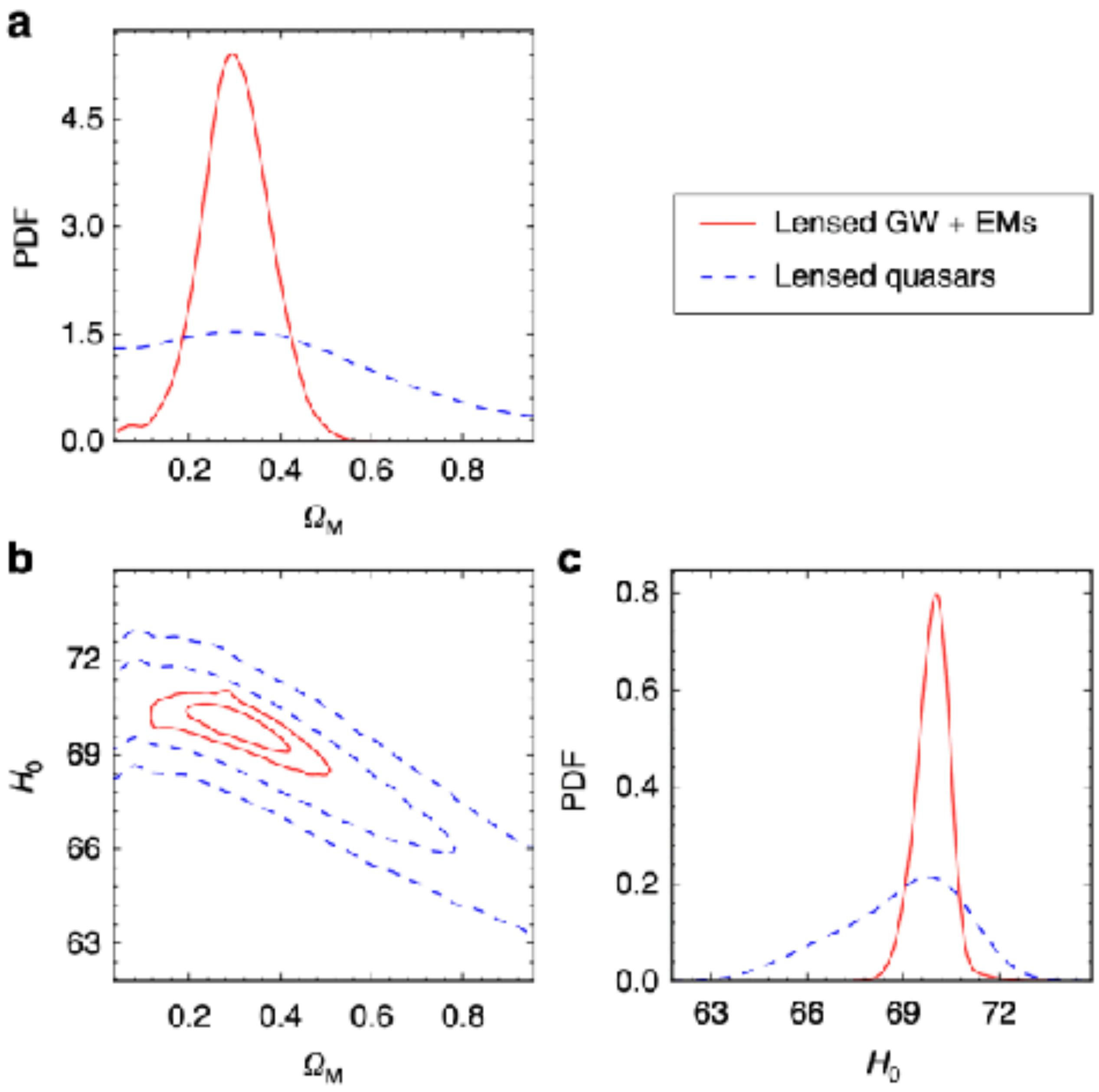
Matteo Bianconi ✉, Graham P Smith, Matt Nicholl, Dan Ryczanowski, Johan Richard, Mathilde Jauzac, Richard Massey, Andrew Robertson, Keren Sharon, Evan Ridley

*Monthly Notices of the Royal Astronomical Society*, Volume 521, Issue 3, May 2023, Pages 3421–3430, https://doi.org/10.1093/mnras/stad673

# Strong Lensing of Gravitational Waves

- Expected numbers
  - LIGO design sensitivity: 1/year
  - Einstein Telescope: ~ 100/year (out of $10^4$-$10^5$ GWs)
- Excellent time delay determination
- Absolute (waveform reconstruction) + Relative strains
- No spatial resolution

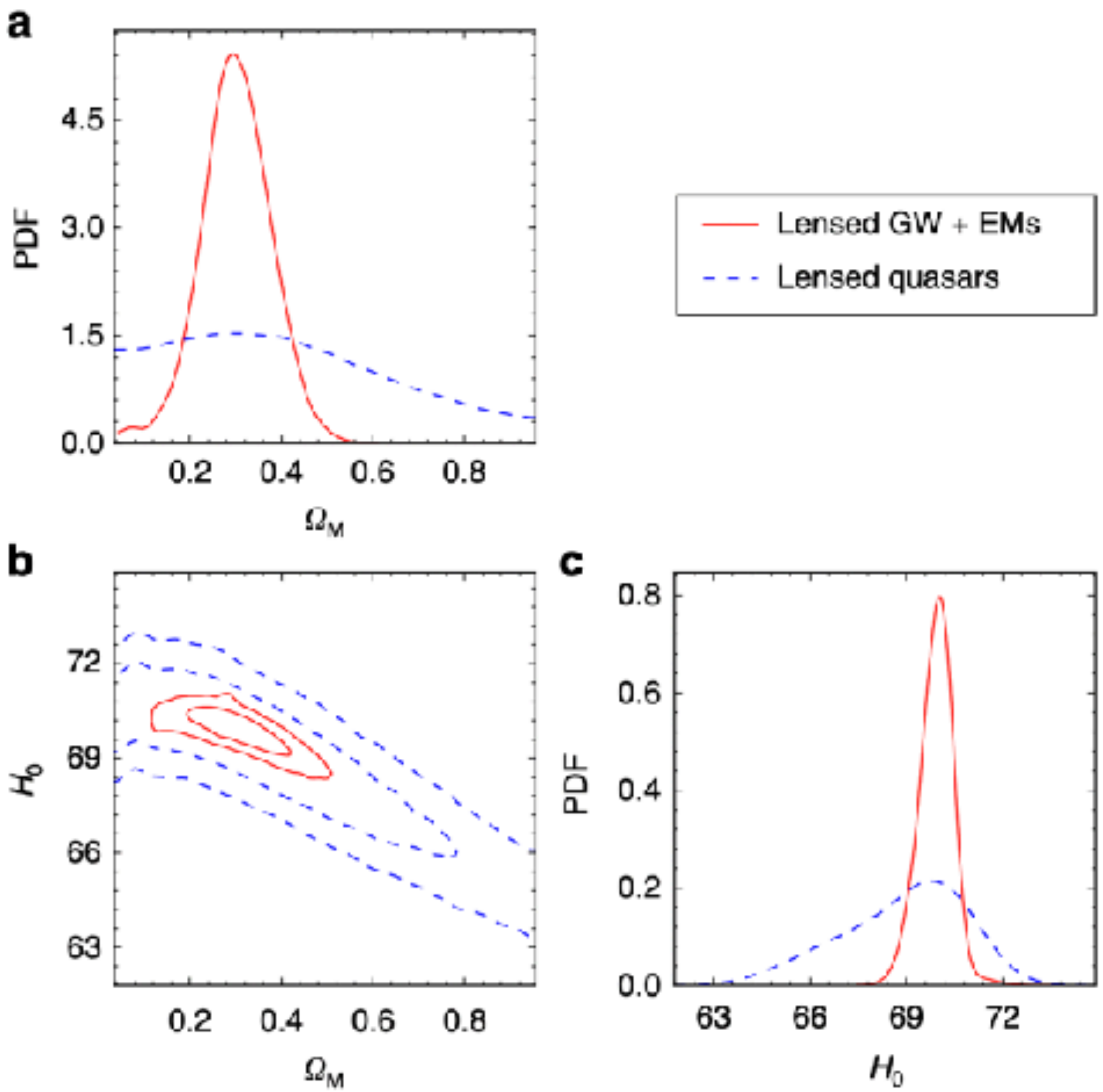
# Lensed GW with EM counterpart: prospects for cosmology



Liao et al., Nature Communications, 8, 1148 (2017)

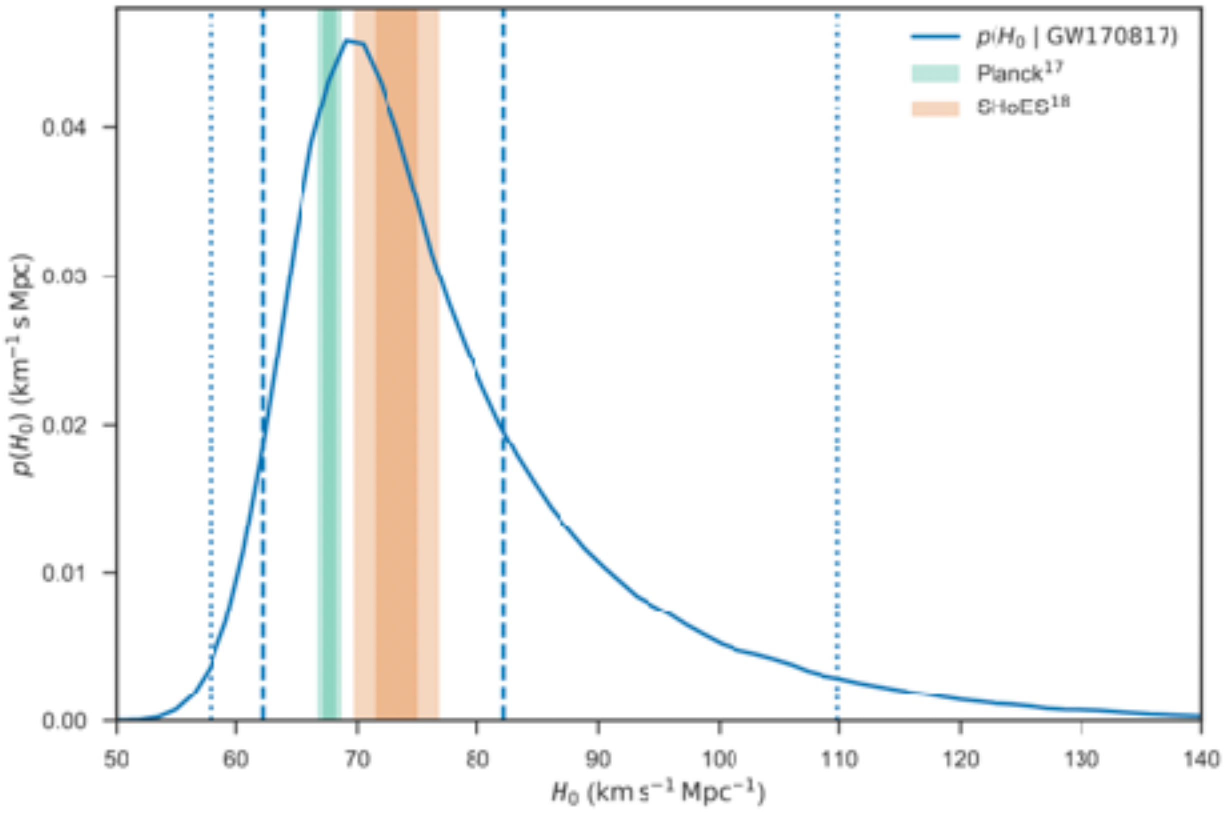
- Typically optical counterparts (MMA)
- Extremely valuable
- Extensive follow-up program
- Only GW170817 for now
- Example not using standard siren:
  - Waveform independent
  - Better reconstruction (transient goes away)
  - < 0.7% determination of $H_0$ from 10 Strongly Lensed BBH!

# Lensed GW with EM counterpart: prospects for cosmology



Liao et al., Nature Communications, 8, 1148 (2017)

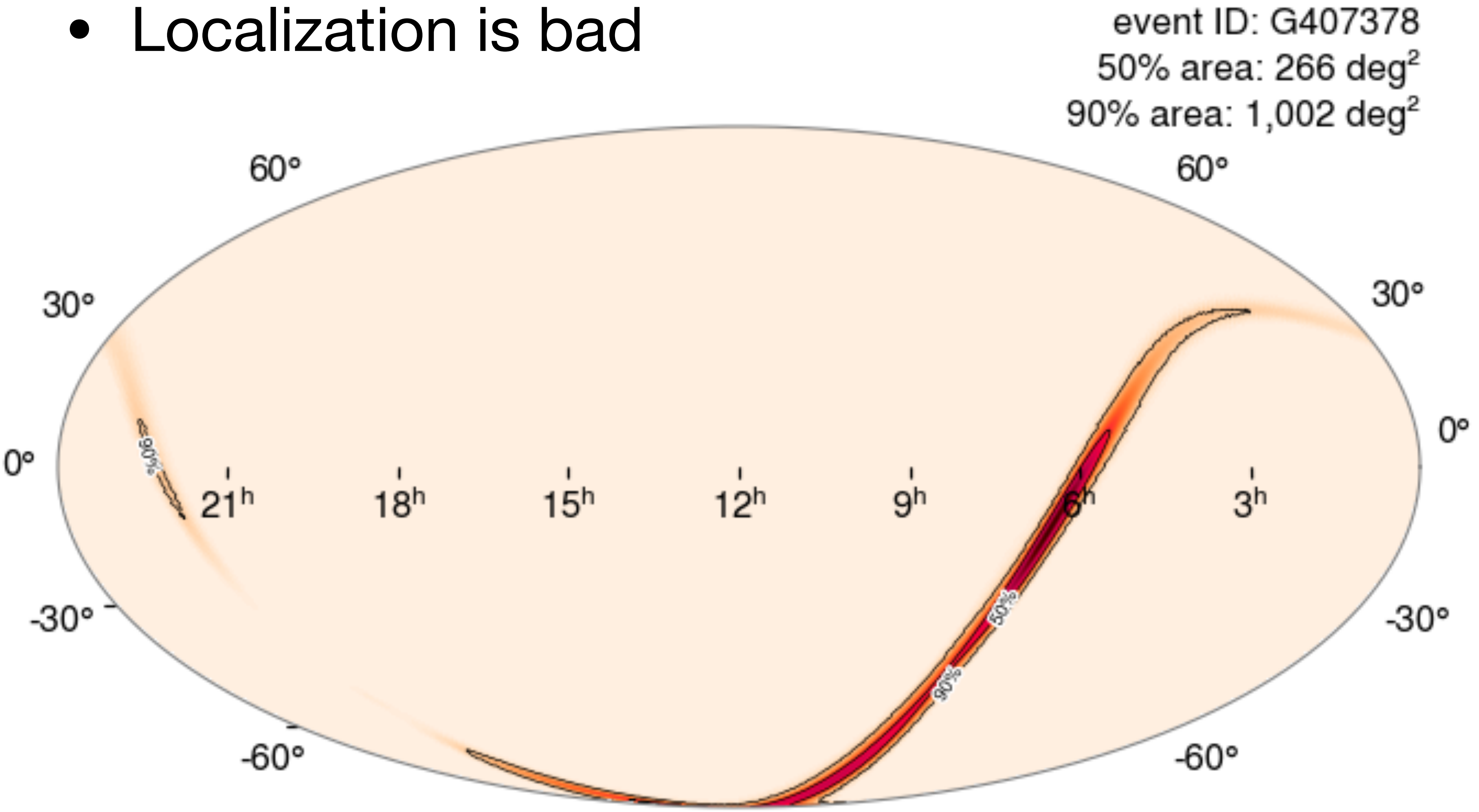
- Compare to GW170817 standard siren (complementary to)

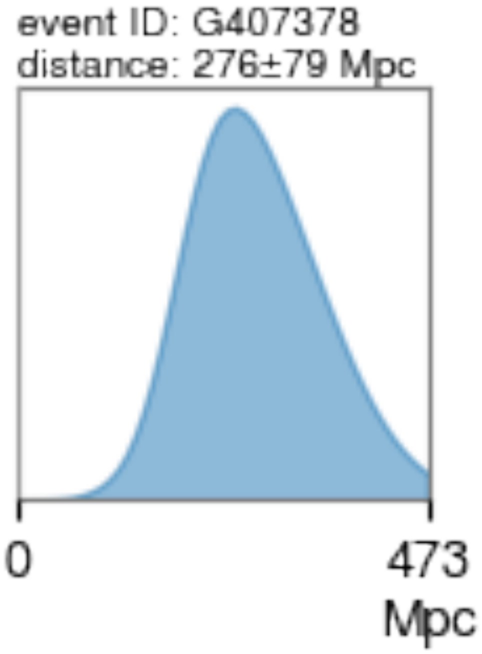


LIGO, Nature 551, 85 (2017); arXiv:1710.05835

# Lensed GW with no EM counterpart

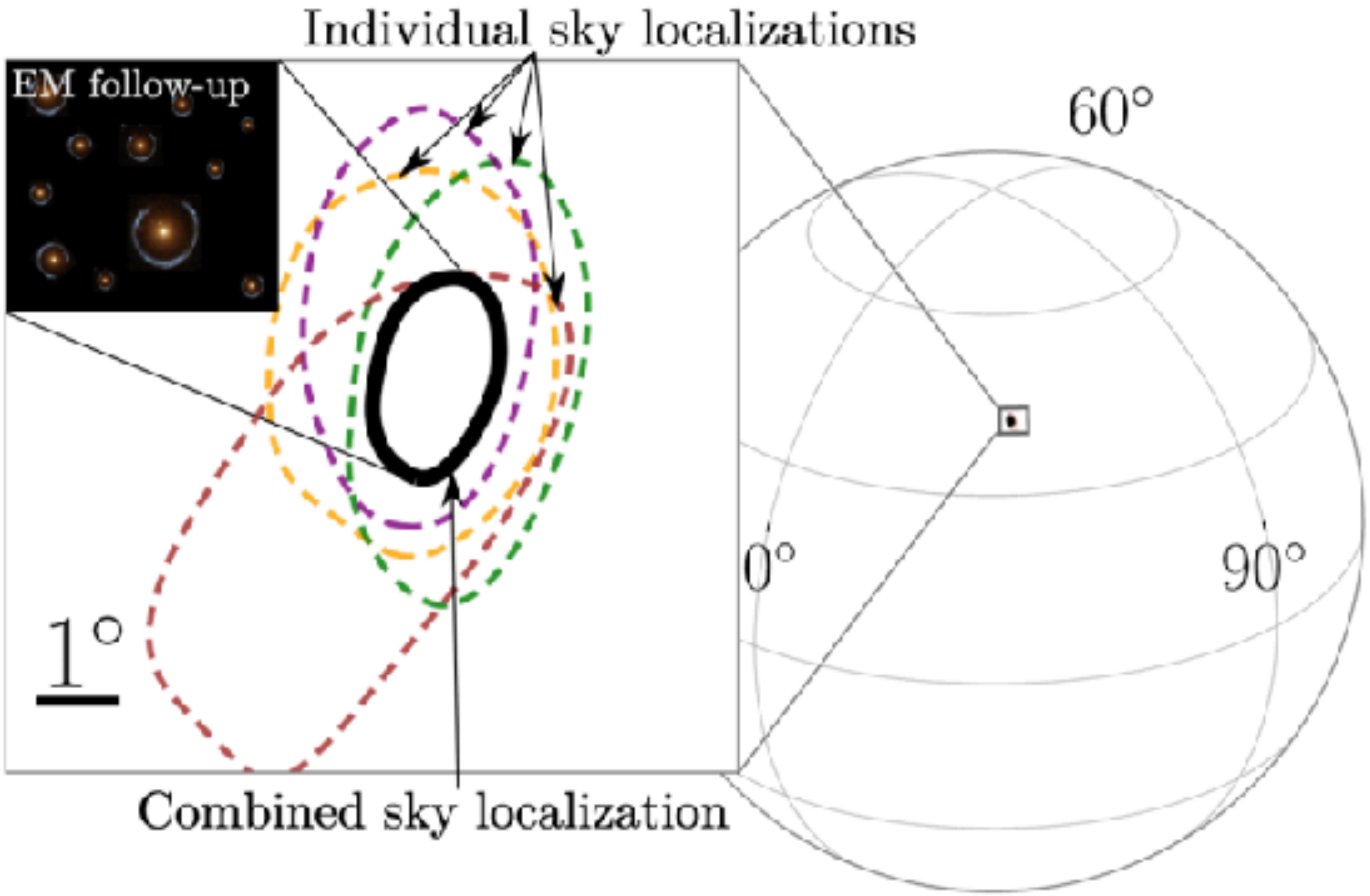
- BBH much more frequent than Kilonovae
- Localization is bad





https://gracedb.ligo.org/superevents/S230518h/

# Localization

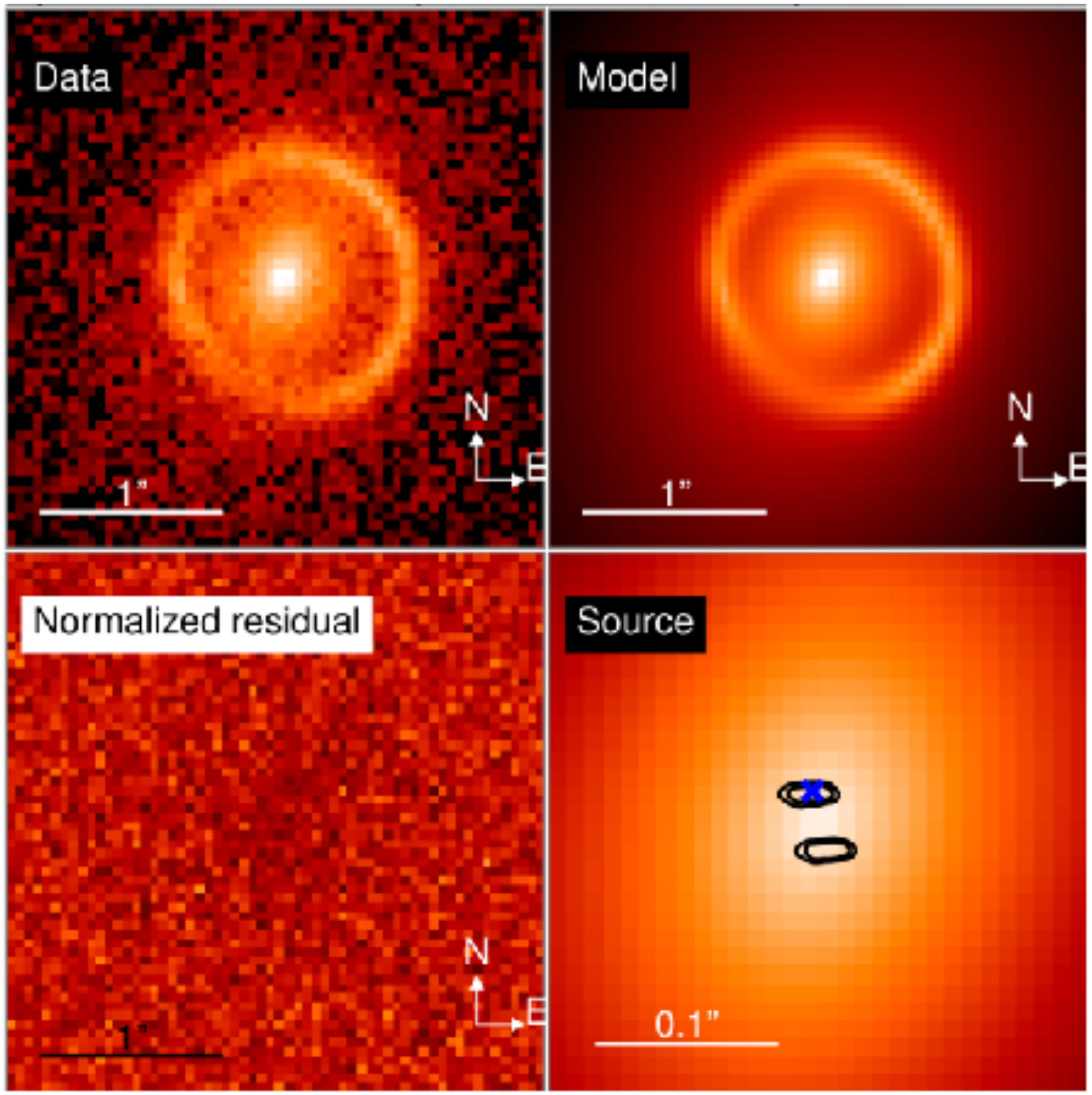


arXiv:2004.13811

- Needs spectroscopic data of both lens and source galaxies
- High resolution imaging of candidate(s)
- Important optical follow-up program (MMA)

- If GW is strongly lensed, its host galaxy should be too!
- Strong Lenses are rare, yet, still ~ 100/sq-deg
- Use relative time-delays and strains to pin-point the right system!
- Use quadruply imaged systems
- Relies on detection of all strongly lensed galaxies to required depth

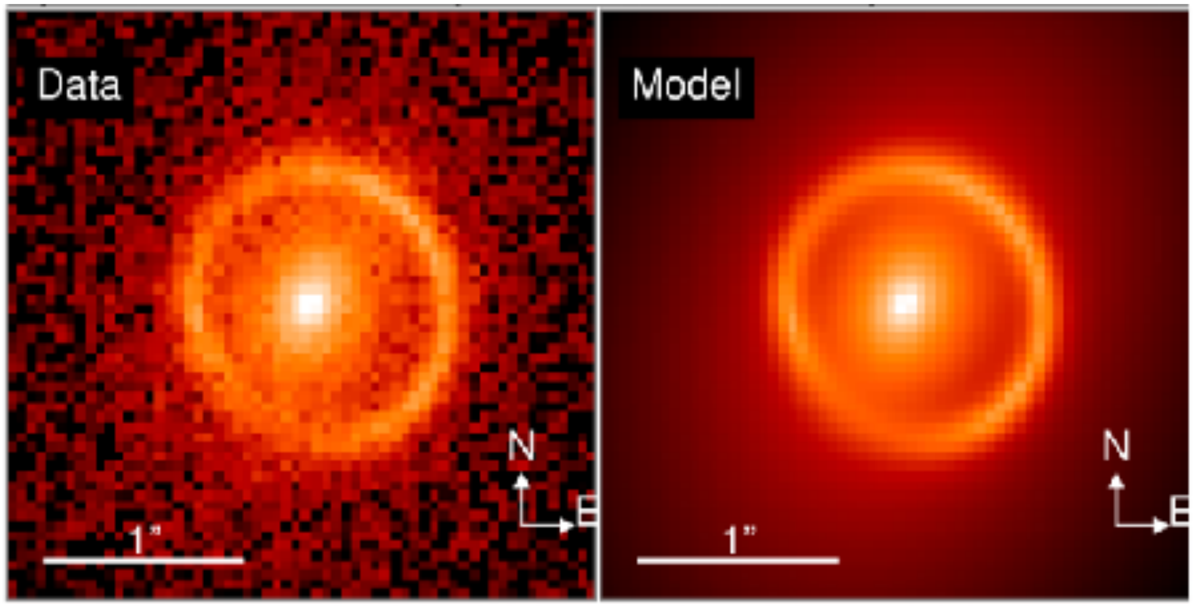
# Lensed Binary Black-Holes

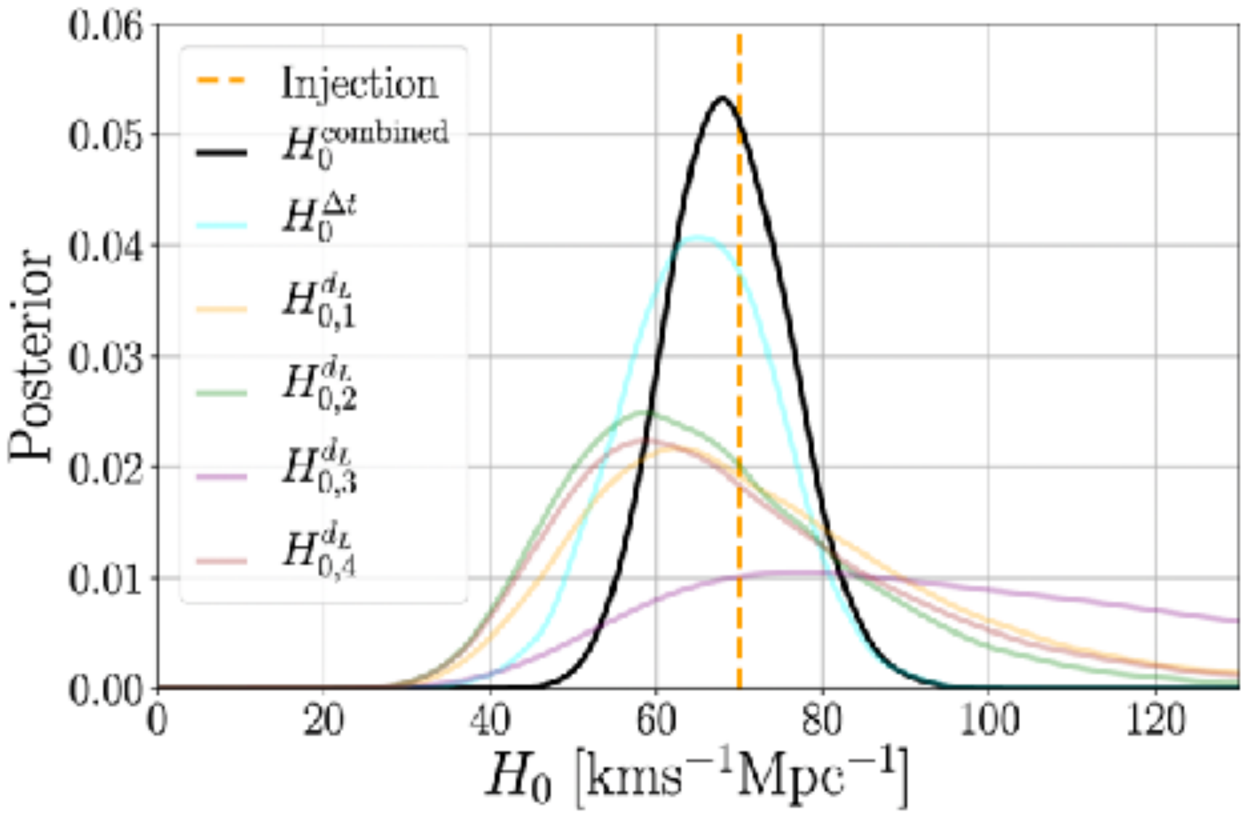


arXiv:2004.13811

- Source and lens potential reconstruction from high resolution imaging
- Extra constraints from relative magnifications and time-delays
- BBH localization within the host galaxy!
- 10% determination of $H_0$ from a single Strongly Lensed BBH!

# Lensed Binary Black-Holes





arXiv:2004.13811

- Source and lens potential reconstruction from high resolution imaging
- Extra constraints from relative magnifications and time-delays
- BBH localization within the host galaxy!
- 10% determination of $H_0$ from a single Strongly Lensed BBH!

# Summary: lensing and MMA

- Microlensing (in the galaxy and beyond)
    - Can complement the census of compact objects (no EM nor binaries needed)
    - Provide constraints on BH on all mass scales (including PBH) + new GR probe
- Strong Lensing
    - Transients and variable sources: stronger cosmological constraints
    - Enables modelling for GW lensing
- Lensing of GW: truly MMA science
    - GW with optical counterparts
    - Dark Sirens: Strong Lensing may localize the source!
- Should be detected in the near future: lots of science!
    - Next generation optical and GW instruments + follow-ups
    - Need to get ready now!

- **A lot of room for contributions on theory, simulations, observations and data analysis!**

Thank you

# Resources

- Comprehensive review on gravitational lensing: *33rd Advanced Saas Fee Course on Gravitational Lensing: Strong, Weak, and Micro*; https://inspirehep.net/conferences/974653
- Excellent resource web-page/portal on microlensing: http://www.microlensing-source.org/

**Strong Lensing by galaxies**

- A. J. Shajib et al., *Strong Lensing by Galaxies*, arXiv: 2210.10790

**Time delays from multiply imaged transients**

- Ding, Xuheng; Liao, Kai; Birrer, Simon; Shajib, Anowar J.; Treu, Tommaso; Yang, Lilan (2021), *Improved time-delay lens modelling and H0 inference with transient sources*, MNRAS 504, 4, 5621; arXiv: 2103.08609

**Lensing of gravitational waves:**

- LIGO-Virgo-KAGRA webinar: Gravitational-wave lensing https://www.youtube.com/watch?v=tBVS12kXJwE, arxiv:2105.06384
- Liao, Fan, Ding, Biesiada & Zhu, Precision cosmology from future lensed gravitational wave and electromagnetic signals, Nature Communications 8, 1148 (2017)
- Diego, J. M. , Broadhurst, T. , Smoot, G. F. , Evidence for lensing of gravitational waves from LIGO-Virgo data; https://ui.adsabs.harvard.edu/abs/2021PhRvD.104j3529D/abstract
- Aleksandra Piórkowska, Silesia U., Marek Biesiada, Zong-Hong Zhu, 2013, Strong gravitational lensing of gravitational waves in Einstein Telescope, JCAP 10 (2013) 022; 1309.5731
- G. Pagano, O. A. Hannuksela, T. G. F. Li, LensingGW: a Python package for lensing of gravitational waves, https://www.aanda.org/articles/aa/pdf/2020/11/aa38730-20.pdf